1. Find the Nash bargaining model NTU solution to the bimatrix game

$$\begin{bmatrix} (0,\ 2) & (2,\ \text{-}1) \\ (\text{-}1,\ \text{-}3) & (\text{-}2,\ 1) \end{bmatrix}$$

with threat point $(-1, -2)$.

2. Let

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 0 & \text{-}1 \\ \text{-}3 & \text{-}1 & 0 \end{bmatrix}$$

be the matrix of a matrix game. (a) Given that $\hat{q} = (1/2, 1/3, 1/6)^T$ is the strategy for Colin (Player II), find the best response for Rose (Player I), showing your work. (b) Calculate an optimal strategy for Rose. (Note: Part (b) has nothing to do with part (a).)

3. Let (N, v) be a 3-person game in coalitional form with characteristic function $v(\emptyset) = 0, v(1) = 1, v(2) = 1, v(3) = 3, v(12) = 2, v(13) = 5, v(23) = 6, v(N) = 10$. (a) Determine the constants $c_S(v)$ to write v in the form

$$v = \sum_{S \subseteq N} c_S(v)\omega_S$$

where $\omega_S(T) = 1$ if $S \subseteq T$ and $\omega_S(T) = 0$ otherwise. (b) Use part (a) to calculate $\phi_2(v)$, the Shapley value of Player 2.

4. Find the TU solution to the game with bimatrix

$$(A, B) = \begin{bmatrix} (0,\ 3) & (2,\ 1) \\ (0,\ \text{-}1) & (1,\ 3) \\ (0,\ 1) & (1,\ 2) \end{bmatrix}$$

and describe the side payment that should be made.

5. Find all the winning moves in the game of Nim with three piles of 17, 21 and 30 chips.

6. Given that $p = (11/20, 4/10, 1/20)^T$ is an optimal strategy for Rose in the matrix game with game matrix

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & -1 & 4 \\ -1 & 4 & -3 \end{bmatrix}$$

find an optimal strategy for Colin.

7. Let $(N = \{1, 2, 3\}, v)$ be a 3-person game in coalitional form such that $v(\emptyset) = 0, v(1) = 1, v(2) = 2, v(3) = 0, v(12) = 5, v(13) = 4, v(23) = 4$ and $v(N) = 10$. Find all real numbers s and t such that $x = (s, 2, t)$ is in the core.

8. Let $A = [a_{ij}]$ be an m-by-n matrix, let $c > 0$ and k be constants and let $K = [k_{ij}]$ be the m-by-n matrix such that $k_{ij} = k$ for all i, j. Let V be the payoff in the matrix game with matrix A if the players choose strategies $\hat{p}$ and $\hat{q}$. Prove that if the players still choose the strategies $\hat{p}$ and $\hat{q}$ in the matrix game with matrix $cA + K$, then the payoff will be $cV + k$.

9. A 3-person game in strategic form is given by the matrices

$$\begin{bmatrix} (1, 1, 1) & (0, 1, 3) \\ (2, 0, -1) & (1, -1, -1) \end{bmatrix} \qquad \begin{bmatrix} (-2, 0, 1) & (-1, 1, 0) \\ (1, 2, -1) & (0, 1, 1) \end{bmatrix}$$

where the left matrix is the payoffs if Player I chooses 1 and the right matrix is the payoffs if Player I chooses 2. The rows correspond to choices of 1 and 2 for Player II and the columns choices of 1 and 2 for player III. Calculate the following values of the characteristic function of the coalitional form of the game: (a) $v(N)$ (b) $v(II)$ (c) $v(I\ \ III)$.

10. (a) Prove that if a game (N, v) is inessential, that is,

$$\sum_{i=1}^{n} v(i) = v(N),$$

then there is only one (individually rational and group rational) imputation.
(b) Given that, if (N, v) is a constant-sum (essential) game, then

$$v(S) + v(N - S) = v(N)$$

for every subset S of N, prove that no imputation is stable, that is, the core is empty.