Total score: 28 points 4 questions, 7 points each

- Write your answers on some papers. Scan as a pdf file. Upload the pdf file as CCLE Assignment Midterm 1 before the end time.
- Open book. But you cannot get any help from other people.
- If you use any result from the lecture, cite the result (e.g., Lemma 1 in Section 1).
- Results from the homework cannot be used. If you use such result, you have to prove it.

1. Suppose that M is a metric space and A is a subset of M such that $A \neq \emptyset, M$. Assume that A and $M\backslash A$ are connected but M is disconnected. Show that A is open and closed in M.

2. Suppose that $f : [1, \infty) \to \mathbb{R}$ is a continuous function such that $|f(x)| \leq 2$ for all $x \in [1, \infty)$. Show that the function $g : [1, \infty) \to \mathbb{R}$, defined by $g(x) = \dfrac{f(x)}{x}$, is uniformly continuous.

3. Suppose that M is a compact metric space.

   (i) Let $v \in M$. Let $g : M \to M$. Suppose that $(g(x_n))_{n\in\mathbb{N}}$ converges to $g(v)$ for all sequence $(x_n)_{n\in\mathbb{N}}$ in M such that $(x_n)_{n\in\mathbb{N}}$ converges to v and $(g(x_n))_{n\in\mathbb{N}}$ converges in M. Show that g is continuous at v.

   (ii) Let $w \in M$. Show that if $f : M \to M$ is bijective and continuous ~~at w,~~ then the inverse $f^{-1} : M \to M$ is continuous at $f(w)$.

4. Let E be a set. For each $n \in \mathbb{N}$, let $f_n : E \to [0, \infty)$. Let $f : E \to [0, \infty)$. Show that if $f_n \to f$ uniformly, then the sequence $\left(\inf\limits_{x\in E} f_n(x)\right)_{n\in\mathbb{N}}$ in $\mathbb{R}$ converges to $\inf\limits_{x\in E} f(x)$.

End of exam