MATH 131BH: FIRST MIDTERM

SPRING 2022

LAST NAME ____________________________

FIRST NAME ____________________________

ID NO. ____________________________

Instructions: Perform the stated tasks while keeping in mind the following points:

- This is an exam in rigorous mathematics which may appear light on content but that is because the grading will pay a very serious attention to mathematical details.
- All of the problems generally ask for self-contained proofs (either explicitly or by default) so if you want to borrow on extraneous facts (from class or homework or the textbook) you have to state and prove those as well. That being said, it is fine to use well known (and important) facts without proof but you still have to state these precisely with all needed conditions. The priority is on the main line of the argument so first address that and then proceed to deal with the asides.
- In your proofs, all the objects that you invoke need to be properly defined and all steps in your logical reasoning need to be fully justified.

All these points will be reflected on in the grading. Each problem is worth the same amount of points.

DO NOT WRITE BELOW THIS LINE!

1 ____________ **2** ____________

3 ____________ **4** ____________

TOTAL SCORE ______________

PROBLEM 1

Let $f, g\colon \mathbb{R} \to \mathbb{R}$ be functions (with domain all of $\mathbb{R}$). Do as follows:

(1) Define the phrase "$\lim_{z\to x} f(z)$ exists."
(1) Define the phrase "g is continuous."
(2) Using your definitions, assume that f is such that

$$\forall x \in \mathbb{R}\colon\ h(x) := \lim_{z\to x} f(z) \text{ exists}$$

and prove that h is continuous.

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(1)

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(2)

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(3)

PROBLEM 2

Let $a < b$ be reals and $f\colon [a,b] \to \mathbb{R}$. Do as follows:

(1) Define what it means for f to be of bounded variation.
(2) Define what it means for f to have a discontinuity of second kind at $x \in (a,b)$.
(3) Assuming f is of bounded variation, give a self-contained proof that f does NOT have discontinuity of second kind at any $x \in (a,b)$.

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(1)

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(2)

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(3)

PROBLEM 3

Let $f, g\colon \mathbb{R} \to \mathbb{R}$ (with $\mathrm{Dom}(f) = \mathbb{R}$) be functions. Do as follows:

(1) Define the phrase "g is the derivative of f."

(2) Prove that there is no f whose derivative is the Dirichlet function

$$1_{\mathbb{Q}}(x) := \begin{cases} 1, & \text{if } x \in \mathbb{Q}, \\ 0, & \text{if } x \notin \mathbb{Q}, \end{cases}$$

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(1)

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(2)

PROBLEM 4

Let $a < b$ be reals and $f\colon [a,b] \to \mathbb{R}$ a function (with $\mathrm{Dom}(f) = [a,b]$). Do as follows:

(1) Define the phrase "f is Riemann/Darboux integrable on $[a,b]$."
(2) Using your definition (and no other facts without proof), show that

$$f \text{ non-decreasing } \Rightarrow f \text{ Riemann/Darboux integrable on } [a,b]$$

(You are given a choice to use either Riemann's or Darboux's approach here.)

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(1)

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(2)

SCRATCH PAPER: Will NOT be looked at