

Total score: 7 points

March Boedihardjo © 2021

- Write your solutions on some papers. Scan as a pdf/jpg file(s). Upload the **pdf/jpg** file(s) as CCLE Assignment Quiz 3 before the end time.
- Open book. But you cannot get any help from other people.
- Unless stated otherwise, if you use any result from the lecture, cite the result (e.g., Lemma 1 in Section 1).
- Results from the homework, discussion or textbook cannot be used, unless being stated in lecture notes. If you use such result and it is not stated in lecture notes, you have to prove it.
- ****Recommended:** After submission, logout and log in CCLE. See if your file is there; download the file you submitted and check if it is the file you intended to submit.**

Avoid the following: Upload wrong file or upload only the first page (unless it is intended to be only 1 page; in this case, it's ok).

- This quiz is graded by Stan Palasek. If you have any question about the quiz, please contact palasek@math.ucla.edu

1. (7 points) For $n \in \mathbb{N}$, define $f_n : [-1, 1] \rightarrow \mathbb{R}$ by $f_n(x) = \frac{1}{n} \sum_{k=0}^n x^k$ for $x \in [-1, 1]$.

- (i) Find the pointwise limit of $(f_n)_{n \in \mathbb{N}}$. No need to justify your answer.
- (ii) Does the restriction $f_n|_{[0,1]} : [0, 1] \rightarrow \mathbb{R}$ converge uniformly as $n \rightarrow \infty$? Justify your answer.
- (iii) Does the restriction $f_n|_{[-1,0]} : [-1, 0] \rightarrow \mathbb{R}$ converge uniformly as $n \rightarrow \infty$? Justify your answer.

You may use the formula $\sum_{k=0}^n x^k = \frac{1 - x^{n+1}}{1 - x}$ for $x \neq 1$ but you are not required to use it.

End of exam