Math 131AH, Honors Analysis, UCLA
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Exam 2, November 14, 2016

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No electronics are permitted. You can use results from the course in your proofs, but please say what results you are using.

(1) If $(s_n)_{n \in \mathbf{N}}$ is a sequence of complex numbers, define its sequence of arithmetic means σ_n to be

$$\sigma_n = \frac{s_0 + s_1 + \cdots + s_n}{n+1}.$$

If $\lim s_n = s$, prove that $\lim \sigma_n = s$.

(2) Let X be a metric space which is a union of two subsets, $X = A \cup B$. Suppose that A and B are connected, and that $A \cap B$ is not empty. Show that X is connected.

(3) Let $e = \sum_{n=0}^{\infty} 1/n!$, as usual. Let N be the smallest natural number such that

$$\left| e - \sum_{n=0}^{N} \frac{1}{n!} \right| < \frac{1}{100}.$$

Give explicit upper and lower bounds for N. (You don't have to find optimal bounds, but try to make your upper bound at most 2 times your lower bound. With care, you may be able to compute N exactly.)

$\underline{Y}$ $e - \frac{1}{1} - \frac{1}{2} - \frac{1}{6} - \frac{1}{24} -$

$\frac{7}{720} \approx \frac{1}{100}$ $\quad \frac{1}{<6^2 \cdot 120}$ $\quad < \frac{1}{6^3 \cdot 120}$

$\frac{1}{120} + \frac{1}{6 \cdot 120} + \frac{1}{6 \cdot 7 \cdot 120} + \frac{1}{6 \cdot 7 \cdot 8 \cdot 120}$