

Math 131AH, Honors Analysis, UCLA  
Fall 2016  
Exam 1, October 17, 2016

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No electronics are permitted. You can use results from the course in your proofs, but please say what results you are using.

- (1) Find a bijection between  $[0, 1]$  and the half-open interval  $[0, 1)$ .
- (2) Let  $a$  and  $b$  be elements of an ordered field  $F$ . Show that if  $a \geq c$  for every  $c < b$ , then  $a \geq b$ .
- (3) Show that for every positive real number  $a$ , there is a positive integer  $N$  such that for all integers  $n \geq N$ , we have  $1/\sqrt{n} < a$ .
- (4) Give an example of an infinite closed subset  $E$  of  $\mathbf{R}$  that is contained in  $[0, 1]$  and has empty interior. Please show that your set  $E$  is closed in  $\mathbf{R}$ ; you don't need to prove the other properties, although they should be true.