MATH 131A, SPRING 2020: MIDTERM 2

Name: ______________________

Student ID Number: ______________________

Signature: ______________________

Instructions:

- **There are 5 problems.**
- **In your solutions, you may use without proof any theorem proven in the book/notes/videos for the class (unless the problem itself asks to prove a certain theorem). State clearly (and correctly) when you are using a theorem.**
- **Unless otherwise stated, you need to justify your answer. Please show all of your work, as partial credit will be given where appropriate, and there may be no credit given for problems where there is no work shown.**
- **You have 24 hours to complete and upload the exam on Gradescope, by 11:59pm (PDT) of Monday, May 18.**
- **Any collaboration is prohibited on the test, and will be considered cheating.**

Problem	**Points**	**Score**
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Problem 1. (20 points) Let $f : \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = x^3 - 2x + 1$. *Using the $\varepsilon - \delta$ definition of continuity*, prove that f is a continuous function.

Problem 2. For each of the following series, determine if it converges or diverges, and justify your answer.

(1) (7 points)

$$\sum_{n=1}^{\infty} \frac{n!\,(2n)!}{(3n)!}.$$

(2) (7 points)

$$\sum_{n=1} (-1)^n \frac{\sin n}{n^2}.$$

(3) (6 points)

$$\sum_{n=1}^{\infty} \frac{3^n}{n^3}.$$

Problem 3. (20 points) Suppose $f : (0,1) \to \mathbb{R}$ is uniformly continuous. Prove that $f : (0,1) \to \mathbb{R}$ is bounded.

Problem 4. Let $(a_n)_{n\in\mathbb{N}}$ be a sequence with $\lim_{n\to+\infty} a_n = 0$ and $a_n > 0$ for all $n \in \mathbb{N}$.

(1) (10 points) Show that there exists a subsequence $(a_{n_k})_{k\in\mathbb{N}}$ so that $\sum_{k=1}^{\infty} a_{n_k}$ converges.

(2) (10 points) Show that there exists a subsequence $\left(a_{n_{k_j}}\right)_{j\in\mathbb{N}}$ of $(a_{n_k})_{k\in\mathbb{N}}$ so that

$$\sum_{j=1}^{\infty} a_{n_{k_j}} \leq \frac{1}{2} \sum_{k=1}^{\infty} a_{n_k}.$$

Problem 5. (20 points) Let $f : [0,1] \to [0,1]$ be an increasing function (that is, for every $x_1 < x_2$ in $[0,1]$ we have $f(x_1) \leq f(x_2)$). Prove that there exists $x_0 \in [0,1]$ such that f is continuous at x_0.