5328

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Math 115AH Midterm 2

(1) (25 points) In each of the following give the desired example or answer the question asked. (You do not need to justify your answer):

-5

(a) Write an explicit injective linear transformation from the $\mathbb{C}^{m\times n}$, the complex $m \times n$ matrices viewed as a real vector space to the real vector space of continuous functions on the closed interval $[0,1]$.

(b) Define an explicit vector space V, field F, and linear operator $T: V \to V$ for each of the following:

(i) $\dim_F V = 2$ and $T: V \to V$ has no eigenvalues.

(ii) $\dim_F V = 3$ and $T: V \to V$ has no eigenvalues.

(iii) $\dim_F V = 4$ and $T: V \to V$ has no eigenvalues.

-3

(c) Let V be a finite dimensional complex inner produce space with orthonormal basis $\mathcal{B} = \{v_1, \dots, v_n\}$ and $g \in V^*$, an arbitrary linear functional on V. Let vector $v = \alpha_1 v_1 + \cdots + \alpha_n v_n$ be a in V. Determine the α_i, $i = 1, \dots, n$, in terms of the dual basis to $\mathcal{B}$ and use this to find the v in V such that $g = \langle\ ,v\rangle$.

(d) Let $V = \mathbb{R}^3$ be the real inner product space under the dot product. Let

$$W_1 = \operatorname{Span}((1,-1,0),(0,1,-1)) \text{ and } W_2 = \operatorname{Span}((1,0,-1),(1,1,1)).$$

Determine $(W_1 + W_2)^\perp$, $W_1^\perp + W_2^\perp$, $(W_1 \cap W_2)^\perp$, and $W_1^\perp \cap W_2^\perp$.

(e) Find an explicit infinite dimensional real inner product space V (i.e., not defined as a span of vectors), a basis for it, and the method to get an orthonormal basis for it.

(2) (20 points) Let $V = \mathbb{R}^3$ be an inner product space under the dot product. Let $v = (2,-1,3)$ in V and T the linear operator on V given by reflection about the plane in V perpendicular to v. Compute $T(64, 45, -89)$. You do not need to multiply any matrices that arise in your computation, but you must evaluate inverses. Write matrices that occur clearly.

(3) (35 points)

(a) State and prove the Approximation Theorem.

(b) State two named theorems about linear transformations (after the Dimension Theorem) and fully state a corollary that follows from one of them.

mono & iso MNT.

(c) State two named theorems about inner product spaces (different from the Approximation Theorem) and fully state a corollary that follows from one of them.

GS & OR.

-10

(4) (20 points) Let V be a vector space over F (~~not necessarily finite~~) and $T: V \to V$ a linear operator. Let $\lambda_1, \dots, \lambda_n$ be distinct eigenvalues of T and $W = E_T(\lambda_1) + \cdots + E_T(\lambda_n)$ (not necessarily finite). Define what it means for W to be a direct sum of the $E_T(\lambda_i)$ and prove directly that $W = E_T(\lambda_1) \oplus \cdots \oplus E_T(\lambda_n)$, i.e., you can not just say this follows as a corollary to a theorem proven in class.

$W = E_T(\lambda_1) \oplus \cdots \oplus E_T(\lambda_n)$

$\Leftrightarrow$ $w = w_1 + \cdots + w_n$ $\quad w_i \in E_T(\lambda_i)$ unique

$E_T(\lambda_i) \cap \sum_{\substack{j=1 \\ j \neq i}}^{n} E_T(\lambda_j) = \{0\}$

WTS $E_T(\lambda_1) \cap \sum_{j=2}^{n} E_T(\lambda_j) = \{0\}$

$Tv = \lambda_1 v = \lambda_2 v_2 + \cdots + \lambda_n v_n$