Math 115AH
Midterm II
December 5, 2014

Name:____________________, ____________________
Please put your last name first and print clearly

Signature:______________________________

1.__________

2.__________

3.__________

4.__________

5.__________

Extra Credit Problem: __________

You can use the theorems from the book or proved in class, but you should indicate which theorems you are using.
V and W are finite dimensional vector spaces over a field F unless otherwise specified. Both V and W are not the zero vector space. You can assume F is a subfield of $\mathbf{C}$.

$T : V \to W$ is linear: $R(T)$ is the range of T and $N(T)$ is the null space of T.

1.

a) Define what it means for λ to be an eigenvalue of an operator $T : V \to V$.

b) If V is an inner product space with inner product $\langle \alpha, \beta \rangle$ and W is a subspace of V, define $W^{\perp}$.

2. Suppose $T : V \to V$ is a linear operator. Suppose $T^3 = 0$ and for some $\alpha \in V$, that $T^2(\alpha) \neq 0$. Show $\alpha, T(\alpha), T^2(\alpha)$ are linearly indpendent.

3. Suppose $\alpha_1, \ldots \alpha_n$ is an orthonormal basis of V and $\alpha, \beta \in V$.

a) If $\alpha = c_1\alpha_1 +, \ldots + c_n\alpha_n$, how can you express the c_i in terms of the $\langle \alpha, \alpha_i \rangle$?

b) Find a formula for $\langle \alpha, \beta \rangle$ in terms of $\langle \alpha, \alpha_i \rangle$ and $\langle \beta, \alpha_i \rangle$

4. Suppose $T : V \to V$ and $T^n = 0$ for some positive $n \in \mathbf{Z}$. Show that if λ is an eigenvalue of T, then $\lambda = 0$.

5. Suppose $W \subseteq V$ is subspace of an inner product space.

a) Find an inclusion relation between $(W^{\perp})^{\perp}$ and W. Do not assume V is finite dimensional.

b) If V is finite dimensional, show $(W^{\perp})^{\perp} = W$. You can use the theorem about the relationship between $\dim W$, $\dim W^{\perp}$ and $\dim V$.

Extra Credit: Suppose V is not necessarily finite dimensional, but $W \subseteq V$ is finite dimensional.

a) Show $V = W + W^{\perp}$. Hint: If $\alpha_1, \alpha_2, \ldots \alpha_n$ is an orthonormal basis of W, you can define a projection from V to W.

b) What is $(W^{\perp})^{\perp} \cap W^{\perp}$?

c) Show $(W^{\perp})^{\perp} = W$.