Math 115A Midterm 2 2016 November 14 Name: Cetorelli, Ty
Please put your last name first and print clearly

UID: 904 <u>868 337</u>

Signature:_

- NO BOOK - NO CALCULATOR -

You can use both sides of the sheet for each problem. If you use extra sheets, write your name on them, and indicate which problem you are treating.

All answers must be stated clearly.

All answers must be justified (except for Problem 1).

Verbiage will be ignored or counted negatively if wrong. Good luck!

- SCORE
1. 2. 6

3. 0 4. 5

3. 5. 7

Total 9

Problem 1. [5 pts]

Answer only by "True" (T) or "False" (F) without justification.

(a) Two finite-dimensional vector spaces with the same dimension are always isomorphic.

False

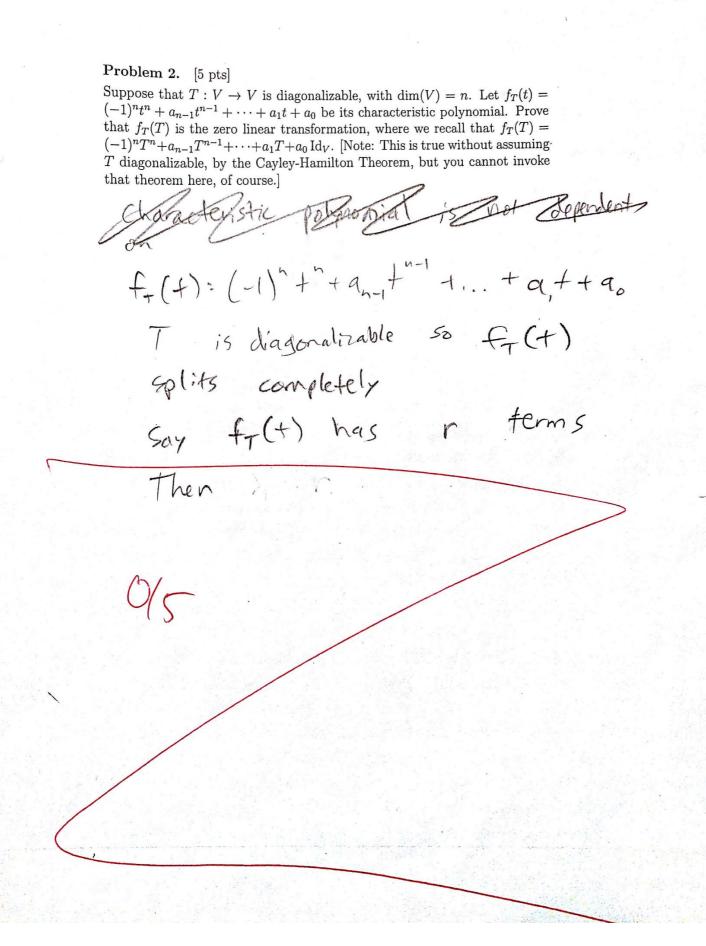
(b) The sum S+T of two linear isomorphisms $S,T:V\to W$ is again a linear isomorphism.

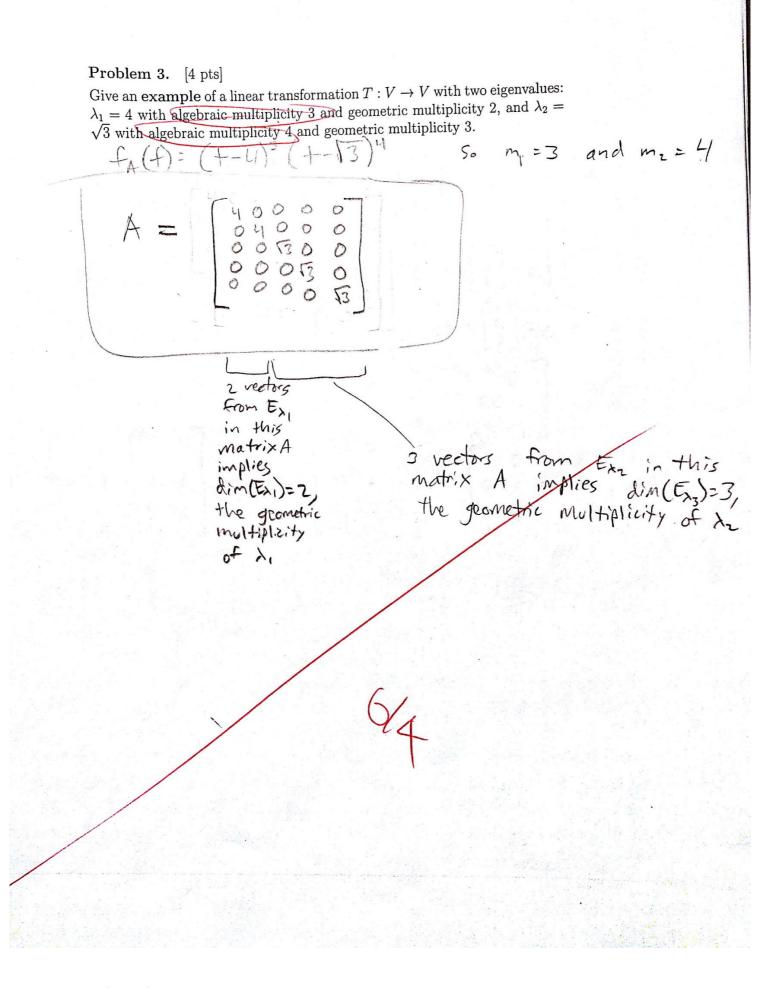
Tool

(c) Consider a square matrix of the form $A = \begin{pmatrix} B & 0 \\ C & D \end{pmatrix}$, with B a square matrix. Then A is invertible if and only B and D are invertible.

(d) A square matrix whose characteristic polynomial splits completely is diagonalizable.

(e) For $A \in M_{n \times n}(\mathbb{R})$ to be diagonalizable, it suffices that the geometric multiplicity of every eigenvalue of A equals its algebraic multiplicity.





Problem 4. [5 pts] Consider the linear transformation $T = L_A : \mathbb{R}^2 \to \mathbb{R}^3$ given by the matrix $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \\ 4 & 2 \end{pmatrix}$. Give the matrix of T with respect to the ordered bases $\mathcal{B} = \left\{ \begin{bmatrix} 1\\2 \end{bmatrix}, \begin{bmatrix} 1\\1 \end{bmatrix} \right\} \text{ and } \mathcal{C} = \left\{ \begin{bmatrix} 1\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\0\\1 \end{bmatrix}, \begin{bmatrix} 1\\1\\2 \end{bmatrix} \right\}.$ T(B) = [-1/2] · [2] = [-1/2] + c[1/2] T(B2) = [3/2]. [] = [4] = az [] + bx [9] + cx [2] $\begin{bmatrix}
1 & 1 & 1 & | & 5 & 4| \\
1 & 0 & 1 & | & 3 & 1| \\
1 & 1 & 2 & | & 8 & 6
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 1 & 1 & | & 5 & 4| \\
0 & 1 & 0 & | & 2 & 3| \\
0 & 0 & 1 & | & 3 & 2|
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 0 & 1 & | & 3 & 1| \\
0 & 1 & 0 & | & 2 & 3| \\
0 & 0 & 1 & | & 3 & 2|
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 0 & 1 & | & 3 & 1| \\
0 & 1 & 0 & | & 2 & 3| \\
0 & 0 & 1 & | & 3 & 2|
\end{bmatrix}$ and $a_1 = -1$ $b_1 = 3 \implies [T(R_1)]_{e} = \begin{bmatrix} \frac{1}{2} \end{bmatrix}$ and This $\left[\begin{array}{c} -7e \\ 7 \end{array} \right] = \begin{bmatrix} 0 & -1 \\ 2 & 3 \\ 3 & 2 \end{array} \right]$

Problem 5. [6 pts]

Prove that the transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 - 2x_3 \\ 2x_1 + x_2 - 2x_3 \\ 4x_1 - 3x_3 \end{bmatrix}$$

is diagonalizable, and diagonalize it.

is diagonalizable, and diagonalize it.

Let
$$A$$
 be the matrix of the linear transformation

 $A = \begin{bmatrix} 3 & 0 & -2 \\ 2 & 1 & -2 \\ 4 & 0 & -3 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 0 & 0 \end{bmatrix}$
 $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 0 & 0 \end{bmatrix}$

A = $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 0 & 0 \end{bmatrix}$

A = $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 0 & 0 \end{bmatrix}$

$$f_{A}(t) = |A - + I_{3}| = |\frac{3-t}{2}| + 0 - \frac{7}{2}|$$

$$= (3-t)|\frac{1-t}{0}| + \frac{7}{2}| + 0 + (-2)|\frac{7}{4}| + 0$$

$$= (3-t)(-3-t+3+t+2-0) + (-2)(0-4+4+1)$$

$$= (3-t)(+2+2t-3) - 8+t8$$

$$= 3+2+6+-9+3-2+2+3+-8+t8$$

$$= -t^{3}+t^{2}+t-1 = -t^{2}(t-1)+1(t-1)$$

$$= (-t^{2}+1)(t-1) = -(t^{2}-1)(t-1)$$

$$f_{A}(t) = -(t+1)(t-1)^{2}$$
Since $f_{A}(t)$ is completely split, A is diagonalizable and, thus, T is too. This is a theorem.

