MIDTERM 1

04/23/2020

Math115A
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Name:

UID:

ATTENTION: **Axiom 4** of the vector space can be used in the following way:

For all $v \in V$, we have that

$$v + (-v) = 0$$

Exercise 1. Let V be a vector space over $\mathbb{R}$. Show that for all $v, w \in V$ and $\lambda \in \mathbb{R}$, we have that

$$\lambda(v + (-w)) + \lambda((-v) + w) = 0$$

(You should only use the axioms of a vector space and that for all $\lambda \in \mathbb{R}$ we have that $\lambda \cdot 0 = 0$. For each step state CLEARLY which axiom or result you are using.)

ATTENTION: Do NOT use $\lambda(-v) = -(\lambda v)$

Exercise 2. Let V be a vector space and assume that $(v_1, v_2, v_3, v_4, v_5) \in V^5$ is linearly independent. Show that

$$(v_1 + v_2,\ v_1 + 2v_2 + v_3,\ v_3 + 2v_4,\ 2v_4 + (-4)v_5,\ v_1 + (-2)v_5) \in V^5$$

is also linearly independent.

Math 115A Midterm 1

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TOTAL POINTS

12 / 12

QUESTION 1

1 Vector space axioms 7 / 7

- ✓ + 1 pts **Axiom 1 correctly**
- ✓ + 1 pts **Axiom 2 correctly**
- ✓ + 1 pts **Axiom 3 correctly**
- ✓ + 1 pts **Axiom 4 correctly**
- ✓ + 1 pts **Brackets correctly**
- ✓ + 1 pts **Use lambda 0=0 correctly**
- ✓ + 1 pts **Axiom 7 correctly**
- + **7 pts** Correct
- + **0.5 pts** Brackets not all correct

QUESTION 2

2 Linear independence 5 / 5

- ✓ + 1 pts **Setup**
- ✓ + 1 pts **Rearrangement of linear combination**
- ✓ + 1 pts **Application of linear independence**
- ✓ + 1 pts **System of equations**
- ✓ + 1 pts **Statement of conclusion**

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Midterm

Exercise 1:

WTS that $\lambda(v+(-w)) + \lambda((-v)+w) = 0$

$$\lambda(v+(-w)) + \lambda((-v)+w) \overset{ax.7}{=} (\lambda v + \lambda(-w)) + (\lambda(-v) + \lambda w)$$
$$\overset{ax\,2}{=} ((\lambda v + \lambda(-w)) + \lambda(-v)) + \lambda w \overset{ax\,1}{=} (\lambda(-v) + (\lambda v + \lambda(-w))) + \lambda w$$
$$\overset{ax\,2}{=} ((\lambda(-v) + \lambda v) + \lambda(-w)) + \lambda w \overset{ax\,7}{=} (\lambda((-v)+v) + \lambda(-w)) + \lambda w$$
$$\overset{ax\,1}{=} (\lambda(v+(-v)) + \lambda(-w)) + \lambda w \overset{ax\,4}{=} (\lambda(0) + \lambda(-w)) + \lambda w$$
$$\overset{\lambda\cdot 0=0}{=} (0 + \lambda(-w)) + \lambda w \overset{ax\,3}{=} \lambda(-w) + \lambda w \overset{ax\,7}{=} \lambda((-w)+w) \overset{ax\,1}{=} \lambda(w+(-w))$$
$$\overset{ax\,4}{=} \lambda(0) \overset{\lambda\cdot 0=0}{=} 0$$

usually 0 on the left. Need to use ax 1 first. No point reduction

Exercise 2:

WTS that $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \in \mathbb{F}$ such that

$\lambda_1(v_1+v_2) + \lambda_2(v_1+2v_2+v_3) + \lambda_3(v_3+2v_4) + \lambda_4(2v_4+(-4)v_5) + \lambda_5(v_1+(-2)v_5) = 0$

only when $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0$

Rearrange int

$$(\lambda_1+\lambda_2+\lambda_5)v_1 + (\lambda_1+2\lambda_2)v_2 + (\lambda_2+\lambda_3)v_3 + (2\lambda_3+2\lambda_4)v_4 + (-4\lambda_4-2\lambda_5)v_5 = 0$$

Since we know $(v_1, v_2, v_3, v_4, v_5)$ is linearly independent.

$$\begin{cases} \lambda_1+\lambda_2+\lambda_5 = 0 & (1) \\ \lambda_1+2\lambda_2 = 0 & (2) \\ \lambda_2+\lambda_3 = 0 & (3) \\ 2\lambda_3+2\lambda_4 = 0 & (4) \\ -4\lambda_4-2\lambda_5 = 0 & (5) \end{cases}$$

$\Rightarrow$ (2) − (1) gives $\lambda_2 - \lambda_5 = 0$ … (6)

2·(3) − (4) gives $2\lambda_2 - 2\lambda_4 = 0$ … (7)

(6) and (7) give $\lambda_4 = \lambda_5$ … (8)

plugging (8) into (5) $-4\lambda_4 - 2\lambda_4 = 0 \Rightarrow \lambda_4 = 0, \lambda_5 = 0$

plugging $\lambda_4 = 0$ into (7) $2\lambda_2 - 2(0) = 0 \Rightarrow \lambda_2 = 0$

plugging $\lambda_2 = 0$ into (3) $0 + \lambda_3 = 0 \Rightarrow \lambda_3 = 0$

plugging $\lambda_2 = \lambda_3 = 0$ into (1) $\lambda_1 + 0 + 0 = 0 \Rightarrow \lambda_1 = 0$

Thus $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0$

and

$(v_1+v_2, v_1+2v_2+v_3, v_3+2v_4, 2v_4+(-4)v_5, v_1+(-2)v_5)$ is linearly independent.

1 Vector space axioms 7 / 7

- ✓ **+ 1 pts** Axiom 1 correctly
- ✓ **+ 1 pts** Axiom 2 correctly
- ✓ **+ 1 pts** Axiom 3 correctly
- ✓ **+ 1 pts** Axiom 4 correctly
- ✓ **+ 1 pts** Brackets correctly
- ✓ **+ 1 pts** Use lambda 0=0 correctly
- ✓ **+ 1 pts** Axiom 7 correctly
- **+ 7 pts** Correct
- **+ 0.5 pts** Brackets not all correct

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Midterm

Exercise 1:

WTS that $\lambda(v+(-w))+\lambda((-v)+w)=0$

$$\lambda(v+(-w))+\lambda((-v)+w) \overset{ax.7}{=} (\lambda v+\lambda(-w))+(\lambda(-v)+\lambda w)$$

$$\overset{ax\,2}{=} ((\lambda v+\lambda(-w))+\lambda(-v))+\lambda w \overset{ax\,1}{=} (\lambda(-v)+(\lambda v+\lambda(-w)))+\lambda w$$

$$\overset{ax\,2}{=} ((\lambda(-v)+\lambda v)+\lambda(-w))+\lambda w \overset{ax\,7}{=} (\lambda((-v)+v)+\lambda(-w))+\lambda w$$

$$\overset{ax\,1}{=} (\lambda(v+(-v))+\lambda(-w))+\lambda w \overset{ax\,4}{=} (\lambda(0)+\lambda(-w))+\lambda w$$

$$\overset{\lambda\cdot 0=0}{=} (0+\lambda(-w))+\lambda w \overset{ax\,3}{=} \lambda(-w)+\lambda w \overset{ax\,7}{=} \lambda((-w)+w) \overset{ax\,1}{=} \lambda(w+(-w))$$

$$\overset{ax\,4}{=} \lambda(0) \overset{\lambda\cdot 0=0}{=} 0$$

□

Exercise 2:

WTS that $\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5 \in \mathbb{F}$ such that

$\lambda_1(v_1+v_2)+\lambda_2(v_1+2v_2+v_3)+\lambda_3(v_3+2v_4)+\lambda_4(2v_4+(-4)v_5)+\lambda_5(v_1+(-2)v_5)=0$

only when $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda_5=0$

Rearrange int

$$(\lambda_1+\lambda_2+\lambda_5)v_1+(\lambda_1+2\lambda_2)v_2+(\lambda_2+\lambda_3)v_3+(2\lambda_3+2\lambda_4)v_4+(-4\lambda_4-2\lambda_5)v_5=0$$

Since we know (v_1,v_2,v_3,v_4,v_5) is linearly independent.

① $\lambda_1+\lambda_2+\lambda_5=0$

② $\lambda_1+2\lambda_2=0$

③ $\lambda_2+\lambda_3=0$

④ $2\lambda_3+2\lambda_4=0$

⑤ $-4\lambda_4-2\lambda_5=0$

$\Rightarrow$ ② − ① gives $\lambda_2-\lambda_5=0$ … ⑥

2 · ③ − ④ gives $2\lambda_2-2\lambda_4=0$ … ⑦

⑥ and ⑦ give $\lambda_4=\lambda_5$ … ⑧

plugging ⑧ into ⑤ $-4\lambda_4-2\lambda_4=0 \Rightarrow \lambda_4=0, \lambda_5=0$

plugging $\lambda_4=0$ into ⑦ $2\lambda_2-2(0)=0 \Rightarrow \lambda_2=0$

plugging $\lambda_2=0$ into ③ $0+\lambda_3=0 \Rightarrow \lambda_3=0$

plugging $\lambda_2=\lambda_3=0$ into ① $\lambda_1+0+0=0 \Rightarrow \lambda_1=0$

Thus $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda_5=0$

and

$(v_1+v_2,\ v_1+2v_2+v_3,\ v_3+2v_4,\ 2v_4+(-4)v_5,\ v_1+(-2)v_5)$ is linearly independent.

2 Linear independence 5 / 5

- ✓ **+ 1 pts** Setup
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