Math 115A
Linear Algebra

Midterm 1

January 30th, 2019

Instructions: You have 50 minutes to complete this exam. There are four questions, worth a total of 40 points. This test is closed book and closed notes. No calculator or any electronic device is allowed.

For full credit show all of your work legibly. Please write your solutions in the space below the questions; indicate if you use srap paper. Don't write on the back of the pages.

Do not forget to write your name and UID in the space below.

Do not engage in any kind of academic dishonesty, including looking at someone else's exam or letting someone else look at your exam. Remember that you are bound by a conduct code!

Name: ____________________________

Student ID number: ____________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

Problem 1.

Indicate whether the following statements are either true or false. Circle your answer.

(a) [1pts.] The map $d : C^\infty\mathbb{R} \to C^\infty\mathbb{R}$ defined by $d(f) = f'$ is linear.

T **F**

(b) [1pts.] The set $\{(1,1,1),(0,1,1),(0,0,1)\}$ is a basis for F^n.

T **F**

(c) [1pts.] There is a linear map $T : M_{2\times3}\mathbb{R} \to \mathbb{R}^2$ so that $T\left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $T\left(\begin{bmatrix} 2 & -2 & 0 \\ 4 & 6 & 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

T **F**

(d) [1pts.] There is a set of $n+1$ linearly dependent vectors in P_nF.

T **F**

(e) [1pts.] There is a subspace W of $M_{3\times3}F$ so that W contains 10 linearly independent vectors.

T **F**

(f) [1pts.] If $\{x_1, \ldots, x_n\}$ is a generating set for V, and $T : V \to W$ is a linear map, then $\{T(x_1), \ldots, T(x_n)\}$ is a generating set for W.

T **F**

(g) [1pts.] The map $T : P_nF \to P_nF$ defined by $T(f) = f + 1$ is linear.

T **F**

(h) [1pts.] There is a vector space V that contains a generating subset with 10 vectors and a linearly independent subset with 15 vectors.

T **F**

(i) [1pts.] If $S_1 \subseteq S_2$ are subsets of a vector space V and S_2 is linearly independent, then so is S_1.

T **F**

(j) [1pts.] A vector space over $\mathbb{R}$ with positive dimension has infinitely many distinct subspaces.

T **F**

Problem 2.

Consider the linear transformation $T : P_2\mathbb{R} \to \mathbb{R}$ defined by $T(f) = f(0) - f(1)$.

(a) [6pts.] Find a basis for $N(T)$.

$0 = a(0)^2 + b(0) + c - a(1)^2 - b(1) - c$

$0 = -a - b$

$a = -b$

basis for $N(T) = \{(x^2 - x), (1)\}$

(b) [2pts.] Extend your basis for $N(T)$ from the previous part of this question to a basis for all of $P_2\mathbb{R}$.

basis for $P_2\mathbb{R} = \{(x^2-x), (1), (x)\}$

(c) [2pts.] Is there some linear map $S : P_2\mathbb{R} \to \mathbb{R}^2$ that is onto and that has the property that $N(S) = N(T)$? Be sure to justify your answer.

$S: P_2\mathbb{R} \to \mathbb{R}^2 \qquad S(f(x)) = (f(0)-f(1), f(0)-f(1))$

$N(S) = N(T)$

No. For $N(S) = N(T)$, in the pair $(a,b) \in \mathbb{R}^2$ either $a=0$, $b=0$, or $a=b$ and these options make it impossible for S to span $\mathbb{R}^2$. So S, can't be onto under the imposed condition.

Problem 3. *10pts.*

Suppose that $T : V \to W$ is a linear transformation, and that $\{T(x_1), \ldots, T(x_n)\}$ is a linearly independent subset of W. Show that $x_1, \ldots, x_n$ is a linearly independent subset of V. Feel free to use anything we have done in class or from the text, unless it is essentially just the statement of this problem.

$\forall T(x) \in \text{span}\{T(x_1) \ldots T(x_n)\}$

$\forall a_1 \ldots a_n \in F$

$T(x) = a_1 T(x_1) + \ldots + a_n T(x_n)$

If $T(x) = 0$, $a_1 \ldots a_n = 0$ due to linear indep.

$0 = 0 \cdot T(x_1) + \ldots + 0 \cdot T(x_n)$

$0 = T(0 \cdot x_1) + \ldots + T(0 \cdot x_n)$

$0 = T(0 \cdot x_1 + \ldots + 0 \cdot x_n)$

Problem 4.

Determine whether the the following subsets of vector spaces are or are not subspaces. Be sure to justify your answer.

(a) [5pts.] $\{f \in C^\infty\mathbb{R} : \forall x \in \mathbb{R}, f(x) \geq 0\}$ as a subset of $C^\infty\mathbb{R}$ (the vector space of infinitely differentiable functions on $\mathbb{R}$) with the standard $\mathbb{R}$-vector space structure.

No, not closed under scalar mult

closed addition

$f, g \in$ subspace

$f+g = f(x)+g(x) = (f+g)(x) \geq 0$ since $f(x), f(y) \geq 0$

$f+g \in$ subspace

scalar mult

if $c \in \mathbb{R}$ $c<0$

$cf = cf(x) < 0$ since $f(x) \geq 0$ $c<0$

$cf \notin$ subspace

(b) [5pts.] $\{A \in M_{n\times n}F : A + A^T = 0\}$ as subspace of $M_{n\times n}F$ with the standard F-vector space structure.

Yes.

closed under addition

for A, B elements of subspace

let $C = A+B$, $C^T = A^T+B^T$

$C+C^T = (A+B)+(A^T+B^T) = (A+A^T)+(B+B^T) = 0$

so, $C \in$ subspace so closed add.

closed under scalar mult

$A \in$ subspace $c \in F$

let $B = cA$, then $B^T = cA^T$

$B+B^T = cA + cA^T = c(A+A^T) = 0$

$B \in$ subspace so closed scalar mult.