Directions: Clearly write your answers on clean paper. Make sure to use complete sentences and justify all of your answers. You can cite theorems from the book using numbers or names.

Remember, you are allowed to access any course materials (the textbook, notes, anything on the CCLE page) while taking the quiz. You are not permitted to seek help from anyone else or search for answers online. You are required to sign an honesty statement acknowledging that you have followed these rules.

When you are done, scan your work using either a smart phone or a scanner.

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1. (Honesty Statement) Write the following statement at the top of your quiz:

   I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this evaluation.

   and then sign your name. **This statement must be written on your quiz with your signature. If the statement or your signature is missing, then your quiz will not be graded.**

2. [5 pts] Let V be a finite dimensional vector space and β be an ordered basis for V. Let $T : V \to V$ be a linear transformation. Use the principle of mathematical induction to prove $[T^k]_\beta^\beta = \left([T]_\beta^\beta\right)^k$ for all nonnegative integers k.

3. Let $\beta = \{1, x, x^2\}$ and $\gamma = \{E^{1,1}, E^{1,2}, E^{2,1}, E^{2,2}\}$ be the standard bases for $P_2(\mathbb{R})$ and $M_{2\times 2}(\mathbb{R})$, respectively. Let $T : P_2(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ be defined via

$$T(a_0 + a_1x + a_2x^2) = \begin{pmatrix} a_0 & -2a_1 \\ -a_2 & a_2 \end{pmatrix}.$$

   (a) [2 pts] Let $p(x) = 1 - 2x + 4x^2$. Compute $[p(x)]_\beta$ and $[T(p(x))]_\gamma$.

   (b) [4 pts] Compute $[T]_\beta^\gamma$.

   (c) [2 pts] Compute $[T]_\beta^\gamma[p(x)]_\beta$ using matrix multiplication. Verify that it equals $[T(p(x))]_\gamma$.

4. [5 pts] Let W be a vector space and let $T : W \to W$ be linear. Prove that $T^2 = T_0$ if and only if $R(T) \subseteq N(T)$. (Recall T_0 denotes the zero transformation.)

5. Let V, W, and Z be vector spaces, and let $T : V \to W$ and $U : W \to Z$ be linear.

   (a) [4 pts] Prove that if UT is one-to-one, then T is one-to-one.

   (b) [3 pts] If UT is one-to-one, then it is not the case that U must one-to-one. Construct an example of transformations U and T where UT is one-to-one, but U is not one-to-one.