Midterm 2
Math 115A-6, Fall 2021

Name	UID	Discussion section

Directions—Please read carefully!

- You are allowed **50 minutes** for this exam. Pace yourself, and do not spend too much time on any one problem.
- No notes, books, your own scratch papers, calculators, cell phones, computers, or other electronic aids are allowed.
- In order to receive full credit, you must **show your work or explain your reasoning.**
- Unless otherwise indicated, please simplify your answers.
- You can use the backs of pages as scratch papers, but **only those written in the front of pages** will be graded.
- Please write neatly. Illegible answers will be assumed to be incorrect. **Circle or box your final answer** when relevant.

Good luck!

Question	Points	Score
1	12	
2	15	
3	20	
4	20	
5	33	
Total:	100	

1. **NO EXPLANATION** is needed for the True and False questions.

(3) (a) True or False: Let V be a finite-dimensional vector space. Let $T: V \to V$ be a linear transformation. If for each eigenvalue of T, the geometric multiplicity is equal to the algebraic multiplicity, then T is diagonalizable.

(3) (b) True or False: Let V be a vector space over the field $\mathbb{R}$ of real numbers. Let $T: V \to V$ be a linear transformation. If v is an eigenvector of T, then a non-zero scalar multiple of v is also an eigevector of T.

(3) (c) True or False: Let $T: V \to V$ be a linear transformation, and β, β' two basis of V. Then $[T]_\beta$ and $[T]_{\beta'}$ are similar matrices.

(3) (d) True or False: Let $T: V \to W$ be a linear transformation. If $\dim V = \dim W$, then T is one-to-one.

(15) 2. Let U, V and W be vector spaces. Let $T: V \to W$ and $S: U \to V$ be linear transformations. Prove that if $T \circ S$ is onto, then T is onto.

(20) 3. Let $\beta = \{\begin{pmatrix}3\\4\end{pmatrix}, \begin{pmatrix}4\\5\end{pmatrix}\}$ be a basis of $\mathbb{R}^2$. Find a matrix $Q \in M_{2\times 2}(\mathbb{R})$ so that the equation

$$[L_A]_\beta = QAQ^{-1}$$

holds for all matrices $A \in M_{2\times 2}(\mathbb{R})$.

(20) 4. Let $T\colon P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be the linear transformation given by

$$T(ax^2 + bx + c) = 2ax^2 + (3b - c)x + 2b.$$

Find all the eigenvalues of T and the algebraic multiplicities of the eigenvalues.

5. Let $T\colon M_{2\times 2}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ be the linear transformation given by

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto -{}^{t}A = \begin{pmatrix} -a & -c \\ -b & -d \end{pmatrix}.$$

We know that T has an eigenvalue -1 with algebraic multiplicity 3 and an eigenvalue 1 with algebraic multiplicity 1.

(23) (a) Find a basis β of $M_{2\times 2}(\mathbb{R})$ consisting of eigenvectors of T.

(10) (b) Write down the matrix $[T]_\beta$, where β is the basis you found in (a). *No explanation is needed, but a wrong answer with some reasonable thoughts written down can earn you partial credits.*