

**NAME:**

**Email:**

**UCLA ID:**

**Sign after the following sentence:** I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this evaluation.

- (1) You have 1 hour. You will receive a 24hr window and you can choose any 1hr in that 24hr window to complete the exam and submit it.
- (2) This is a non-collaborative open-book exam.
- (3) There are a total of 4 problems.
- (4) You need to justify all answers.
- (5) You may take a printout and write your solutions in the space below the questions; if you need more space use the back of the page or use extra sheets. You may also type your answers on a document and submit that. You may also write your answers on blank pages and scan and submit them. The size limit is 100MB for file submissions, so if your scanned file is bigger than that, use multiple files. (If you scan at a reasonable DPI, you should not encounter any problems.)
- (6) Do not forget to write your details and sign the pledge in the space above; if you are not using this exam booklet to submit your answer, be sure to write the above details and write out the above pledge.

MATH 100 Midterm  
2020-11-04

- (Q-1) Let  $a, b > 0$  be real numbers satisfying  $a^8 + b^8 = 2$ . Prove  $a + b \leq 2$ .
- (Q-2) Let  $S$  be a set of  $n$  positive integers. Prove that there exists a non-empty subset of  $S$  such that the sum of its elements is divisible by  $n$ .
- (Q-3) (a) How many zeros does the decimal expansion of  $2020!$  end with?  
(b) What are the last three digits of  $3^{1605}$ ?
- (Q-4) (a) Find a polynomial  $P(x) \in \mathbb{Q}[x]$  such that  $(x^2 + 1)P(x) + 1$  is divisible by  $(x^3 + 1)$ .  
(b) Prove that there is no such polynomial  $P(x) \in \mathbb{Z}[x]$ .
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