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Midterm
LING 180/208
Fall 2016

Name: [redacted]

Instructions

1. No collaboration, you may only consult with Jos Tellings or Deborah Wong.
2. Due on Thursday October 27, beginning of class.
3. Please submit your answers together with this exam.
4. Exercise 10 is required for 208 students, extra credit for 180 students.

Total points for 180 students: 82 points (max score with bonus = 90 points)
Total points for 208 students: 90 points

Exercises

1. (10 points) **Set Theory**

(a) (4 points) Consider the sets $X = \{2,4\}$, $Y = \{1,4\}$, and $Z = \{2\}$. Write the sets $X \cap Y$, $Z - X$, and $X \times Y$. Describe in words what the elements of Z^* look like.

(b) (3 points) Give an example of three sets A, B, C such that $A \cap B \cap C = \emptyset$, but $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$ and $A \cap C \neq \emptyset$.

(c) (3 points) Write $\subseteq$, considered as a relation on $\mathcal{P}(\mathrm{N})$, in set-abstraction notation. Do not use the symbol '$\subseteq$' in your answer!

2. (10 points) **Infinity**

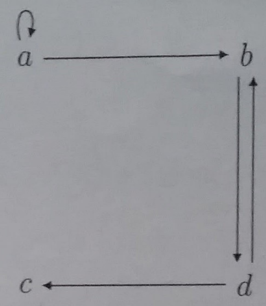
(a) (6 points) Let S be the set $\mathrm{N} - \{2,3\}$. Show that $\mathrm{N} \approx S$:
   i. by using our original definition of $\approx$ (top of page 22).
   ii. by using the Schröder-Bernstein Theorem (Theorem 2.2 on page 46).

(b) (4 points) Define *recursively* a one-to-one function F from N into the set of grammatical English sentences.
   i. Compute $F(3)$ step by step.
   ii. Explain how this constitutes an argument that the set of grammatical English expressions is infinite.

3. (12 points) **Relations**

Part 1

Consider the set $A = \{a, b, c, d\}$ and a binary relation R on A as follows:



In this diagram, a pointed arrow from x to y indicates that $\langle x, y\rangle \in R$. For example, aRa, aRb, etc.

(a) (4 points) The relation R is not symmetric, not asymmetric, not antisymmetric, and not irreflexive. For each of these 4 properties, give a reason why it does not hold for R.

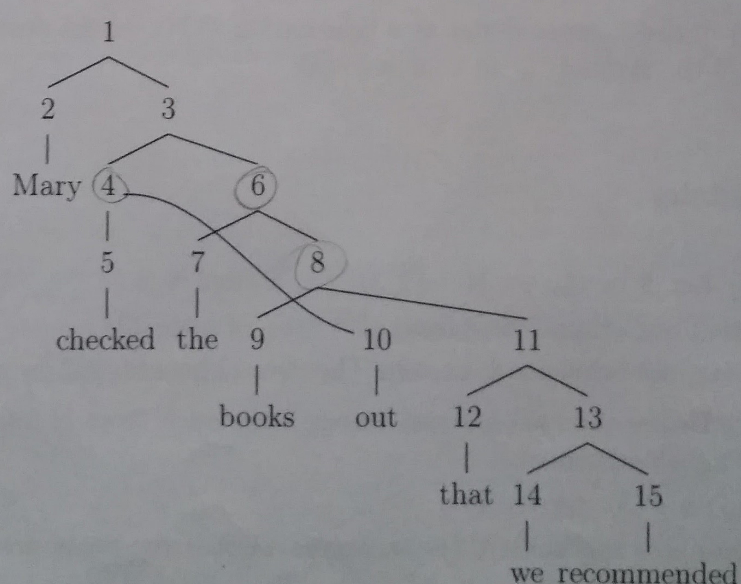
(b) (3 points) The relation R is not transitive. Add the minimum number of arrows needed to make R transitive.

Part 2 (unrelated to the figure)

(c) (5 points) Let B be a set, and S a binary relation on B. Prove: if S is antisymmetric and irreflexive, then S is asymmetric.

4. (9 points) **Trees**

Consider the following tree:



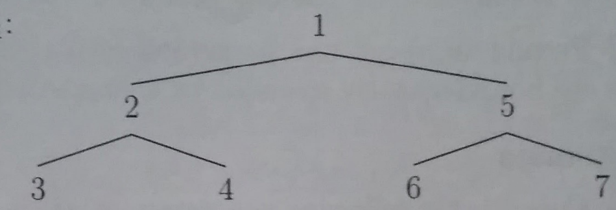
(a) (3 points) Give an example of nodes a and b such that aCCb and bCCa. Give an example of nodes x and y such that x asymmetrically c-commands y.

(b) (3 points) List every node that generates a discontinuous constituent (as defined in Definition 4.19).

(c) (3 points) Explain why the tree does not satisfy the Exclusivity Condition.

5. (10 points) **Isomorphic trees**

Consider the simple tree T_1:



(a) (4 points) Give two other seven-node trees T_2 and T_3 with the property that no two of the trees in $\{T_1, T_2, T_3\}$ are isomorphic.

(b) (6 points) For each of the three pairs of trees (T_1 and T_2; T_1 and T_3; T_2 and T_3), give a sufficient reason why they are not isomorphic.

6. (12 points) **Derivation trees**

(a) (6 points) Using the grammar from Chapter 5, draw derivation trees (can be standard trees) for the following sentences:

1. Dana's brother and some professor congratulated John.
2. No student criticized every professor.
3. Each policeman who Kim praised and congratulated t cried.

(b) (6 points) The following sentence contains an *appositive* relative clause:

(1) Kim, who praised John, laughed.

Assume that appositive relative clauses contain a new lexical item who$_{\text{app}}$.

i. Determine the category of who$_{\text{app}}$ on the basis of the sentence above.
ii. Draw the standard tree of sentence (1).
iii. For each new category that you have used in order to derive (1), say if it combines Functor First or Functor Last with its argument category.

7. (6 points) **Context-free grammars**

Let G be the grammar with $N = \{\text{S}, \text{B}\}$, $T = \{\text{a}, \text{b}\}$, start symbol S, and $\rightarrow$ defined as follows:

S $\rightarrow$ aS | BaB

B $\rightarrow$ b | Bb

(a) (3 points) Write $L(G)$ in set notation.

(b) (3 points) Write a regular expression that corresponds with $L(G)$.

8. (7 points) **Context-free languages**

(a) (3 points) In chapter one we considered the set of DPs $A = \{$the president, the mother of the president, the mother of the mother of the president, ... $\}$. The set A can be matched with $\mathbb{N}$ by the function

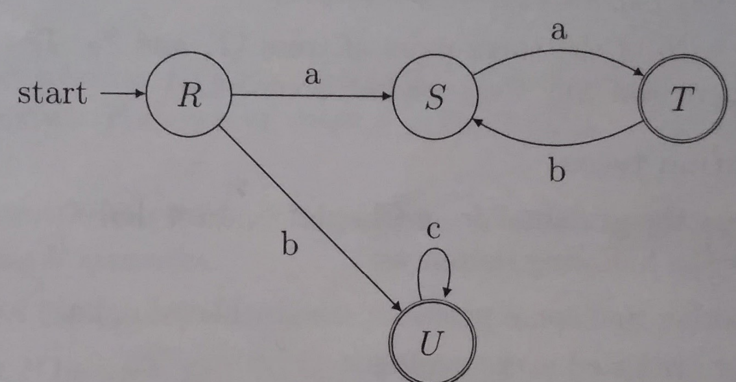
$$F(n) = (\text{the mother of})^n \text{ the president} \qquad \text{for each } n \in \mathbb{N}.$$

Show that A is context-free. Assume that all words are terminal symbols.

(b) (4 points) Present in about one paragraph of text an argument for why natural languages are not adequately modeled by context-free grammars.

9. (6 points) **Automata**

(a) (3 points) Consider the following non-deterministic automaton A over the alphabet {a,b,c}:



What is the language generated by A? (you may give it as a set, or as a regular expression)

(b) (3 points) Give a finite-state automaton over the alphabet {a,b,c} that generates the language of words of even length that begin with abc and end in c.

Extra credit exercise (required for 208 students)

10. (8 points) **Regular languages**

(a) (4 points) Chapter 7 introduces the function $\mathcal{L} : RE(A) \to \mathcal{P}(A^*)$ that assigns to each regular expression r (over an alphabet A) the language that r denotes. Is $\mathcal{L}$ a one-to-one function? Is $\mathcal{L}$ onto? For both, explain why or why not.

(b) (4 points) Write $\mathcal{L}(RE(A))$ for the set $\{\mathcal{L}(r) \mid r \in RE(A)\}$. Show that $\bigcup \mathcal{L}(RE(A)) = A^*$.

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Midterm

a. $X = \{2,4\}$ $Y = \{1,4\}$ $Z = \{2\}$

$X \cap Y = \{2,4\} \cap \{1,4\} = \{4\}$
$Z - X = \{2\} - \{2,4\} = \emptyset$
$X \times Y = \{2,4\} \times \{1,4\} = \{\langle 2,1\rangle, \langle 2,4\rangle, \langle 4,1\rangle, \langle 4,4\rangle\}$
The elements of Z^* are all the finite sequences that consist of the elements of Z. Because $Z = \{2\}$, Z^* will consist of sequences of n repititions of 2, where n is any natural number, as well as the empty sequence e.

$\frac{9}{10}$

b. $A = \{1,3\}$, $B = \{1,2\}$, $C = \{2,3\}$

$A \cap B \cap C = \{1,3\} \cap \{1,2\} \cap \{2,3\} = \{1\} \cap \{2,3\} = \emptyset$
$A \cap B = \{1,3\} \cap \{1,2\} = \{1\}$
$B \cap C = \{1,2\} \cap \{2,3\} = \{2\}$
$A \cap C = \{1,3\} \cap \{2,3\} = \{3\}$

c. $\subseteq =_{df} \{\langle A, B\rangle \mid A, B \in \mathcal{P}(\mathbb{N})$ and for every $x \in A, x \in B\}$
$\langle A,B\rangle \in P(\mathbb{N}) \times P(\mathbb{N})$

2. a. $S = \mathbb{N} - \{2,3\}$
Show that $\mathbb{N} \approx S$.

i. Proof: To show that $\mathbb{N} \approx S$, show that $\mathbb{N} \preceq S$ ① and $S \preceq \mathbb{N}$ ②

① Show that $\mathbb{N} \preceq S$ by showing that there is a one-to-one function from $\mathbb{N}$ into S.
There exists a function $f: \mathbb{N} \to S$ such that

$f(0)=0$, $F(1)=1$, and for all $n \geq 2$, $f(n)=n+2$
Such a function is one-to-one because for every distinct $x, x' \in \mathbb{N}$, $f(n) \neq f(n')$.
Because there exists a one-to-one function from $\mathbb{N}$ to S, $\mathbb{N} \preceq S$.

② Show that $S \preceq \mathbb{N}$ by showing that there is a one-to-one function from S to $\mathbb{N}$.
There exists a function $f: S \to \mathbb{N}$ such that $f(0)=0$, $f(1)=1$, and for all $n \geq 4$, $f(n)=n-2$. Such a function is one-to-one because for every distinct $x, x' \in S$, $f(n) \neq f(n)$.
Because there exists a one-to-one function from S to $\mathbb{N}$, $S \preceq \mathbb{N}$.

Therefore because $\mathbb{N} \preceq S$ and $S \preceq \mathbb{N}$, $\mathbb{N} \approx S$. □

$\frac{10}{10}$

ii. Proof: To show that $\mathbb{N} \approx S$, show that there is a bijection from $\mathbb{N}$ to S. There is a function $f: \mathbb{N} \to S$ such that $f(0)=0$, $f(1)=1$, and for all $n \geq 2$, $f(n)=n+2$.
Such a function is one-to-one because every distinct $x, x' \in \mathbb{N}$ maps to a distinct $f(x), f(x') \in S$. It is onto because the range of f ($\{0,1,4,5,\ldots\}$) is equal to S ($\{0,1,4,5,\ldots\}$). f is therefore a bijection and therefore $\mathbb{N} \approx S$. □

b. Let

$F(n)$ = He is $G(n)$.

$G(0)$ = John, and for all $n > 0$,

$G(n)$ = <the ~ father ~ of ~ $G(n-1)$>

i. $F(3) =$ He is $G(3)$.

$G(3) = \langle$the-father-of-$G(2)\rangle$

$G(2) = \langle$the father-of-$G(1)\rangle$

$G(1) = \langle$the father-of-$G(0)\rangle$

$G(0) =$ John

$G(1) =$ the father of John

$G(2) =$ the father of the father of John

$G(3) =$ the father of the father of the father of John

$F(3) =$ He is the father of the father of the father of John

ii. You can show that a set is infinite if it has a subset that is infinite. The sentences defined by $F(n)$ are a subset of the set of all grammatical English sentences. By defining a bijection F from $\mathbb{N}$ to a subset A of all grammatical English sentences, we are showing by the Schroder-Bernstein theorem that the subset is infinite, and therefore that the set of all grammatical English sentences is infinite. Note that F is a bijection because for every distinct $x, x' \in \mathbb{N}$, $F(x) \neq F(x')$ and the range of F ({He is John, He is the father of John, ...}) is equal to A ({He is John, ...}).

a. 1. R is not symmetric because it is not the case that for all $x, y \in A$, if xRy, then yRx. For example, aRb but $\neg bRa$

2. R is not asymmetric because it is not the case that for all $x, y \in A$, if xRy, then $\neg yRx$. For example bRd and dRb.

3. R is not antisymmetric because it is not the case that for all $x,y \in A$, if xRy and yRx, then $y=x$. For example, bRd and dRb but $b \neq d$.

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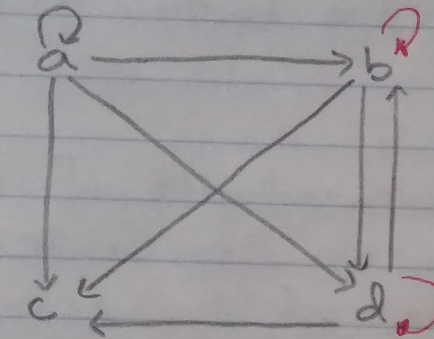
4. R is not irreflexive because it is not the case that for all $x \in A$, $\neg xRx$. For example, aRa.

b.



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c. Prove that if S is antisymmetric and irreflexive, then S is asymmetric.

<u>Proof</u>: Assume S is antisymmetric and irreflexive. Show that S is asymmetric.
Assume by contradiction that S is not asymmetric. This means that it is not the case that for all $x,y \in A$, if x y, then $\neg y$ x, meaning that for some $x,y \in A$, xSy and ySx. Because S is antisymmetric, $x=y$. However, because $x=y$, then xSx, which contradicts the fact that S is irreflexive (for all $x \in A$, $\neg xSx$). Because negating the conclusion leads to a contradiction, S must be asymmetric. □

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a. $a = 14$, $b = 15$
$x = 12$, $y = 14$

b. 4, 6, 8

c. The tree fails the Exclusivity Condition because it is not the case that for all nodes b, d, if b and d are independent then $b <^* d$ or $d <^* b$. For example, 4 and 6 are independent, but it is not the case that every leaf which 4 dominates precedes every leaf which 6 dominates or vice versa. 4 dominates 5 and it precedes 7 and 9 (which 6 dominates), but 4 also dominates 10, which does not precede 7 and 9.

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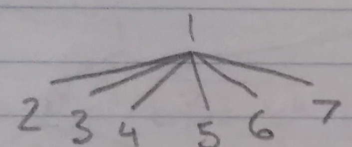
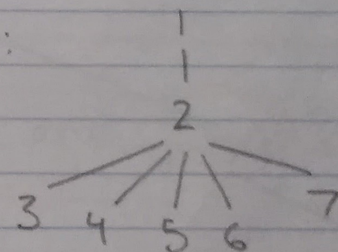
5. a. T_2:



T_3:



10/10

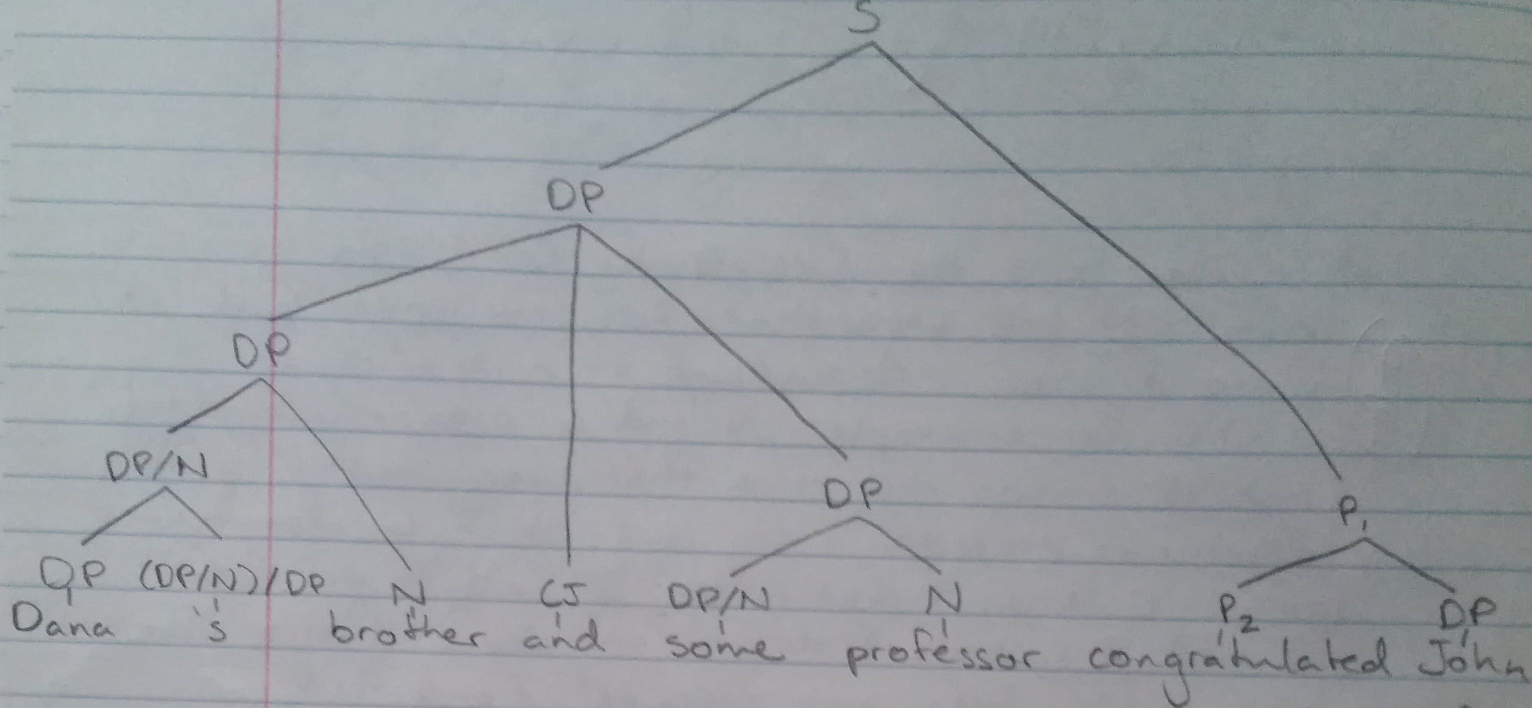
b. T_1 and T_2 are not isomorphic because T_2 contains a node x with deg(x) = 6, and T_1 does not contain such a node.

T_1 and T_3 are not isomorphic because T_3 contains a node x with deg(x) = 5, and T_1 does not contain such a node.

T_2 and T_3 are not isomorphic because T_2 contains a node x with deg(x) = 6, and T_3 does not contain such a node.

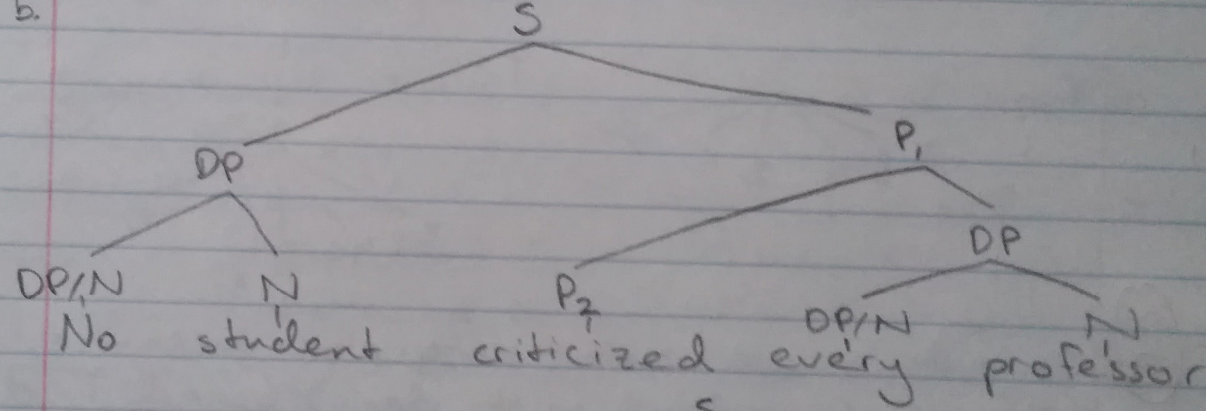
6.

a.

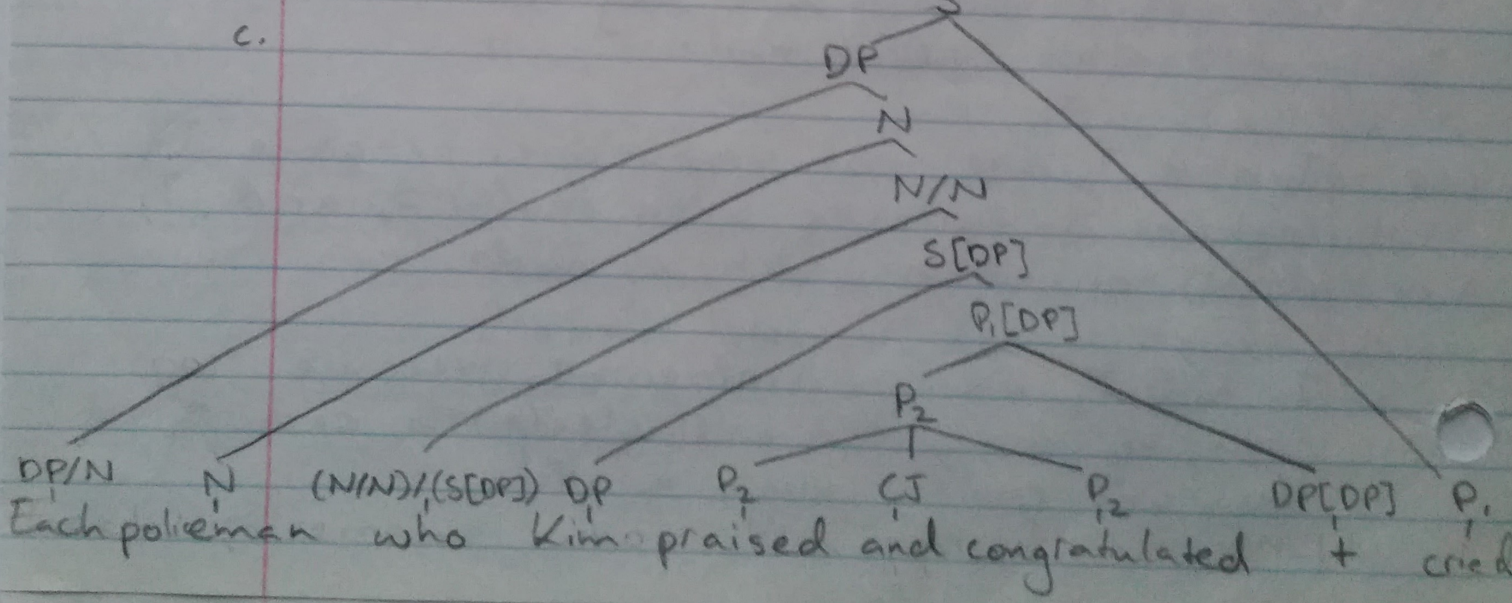

S
DP
DP
DP/N
DP (DP/N)/DP N CJ DP/N N P2 DP
DP
P1
Dana 's brother and some professor congratulated John

b.

6/6.

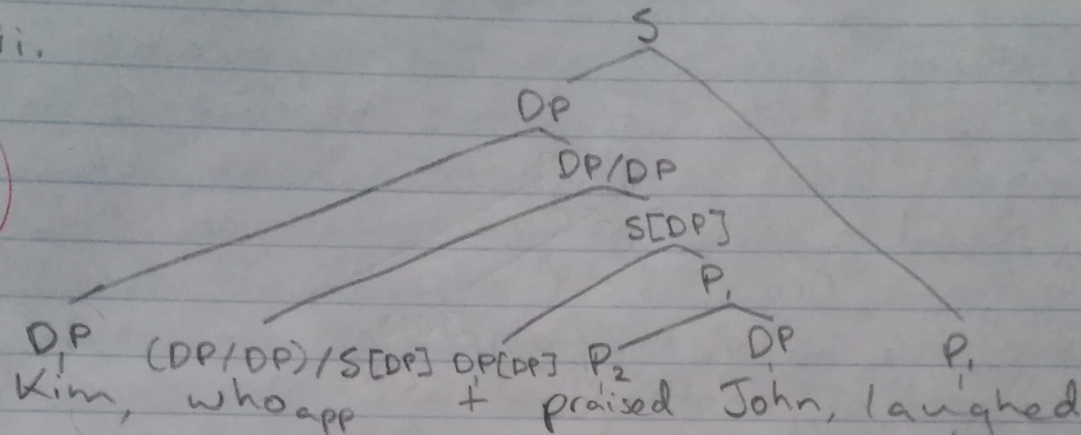

S
DP
P1
DP
DP/N N P2 DP/N N
No student criticized every professor

c.


S
DP
N
N/N
S[DP]
P1[DP]
P2
DP/N N (N/N)/(S[DP]) DP P2 CJ P2 DP[DP] P1
Each policeman who Kim praised and congratulated t cried

b. i. who_{app} has category (DP/DP)/(S[DP]) because it combines with a S[DP] to form a DP/DP which then combines with DP to form a DP.

ii.



iii. (DP/DP)/S[DP] combines with S[DP] functor first. DP/DP combines with DP functor last.
~~Fst~~ ok!

$\frac{6}{6}$

7. a. $L(G) = \{a^i b^j a b^k \mid i \geq 0$ and $j, k \geq 0\}$

$\frac{5}{6}$

b. $\underline{a}^* \cdot (\underline{b} \cdot \underline{b}^*) \cdot \underline{a} \cdot (\underline{b} \cdot \underline{b}^*)$

8. a. Show that A is context-free.

Proof: To show that A is context-free, show that there is a context-free grammar G such that $L(G) = A$. Let $G = \langle N, T, S, \rightarrow \rangle$, where $N = \{S\}$ $T = \{$the, mother, of, president$\}$ and $\rightarrow = \{\langle S$, the mother of $S\rangle, \langle S$, the president$\rangle\}$. By definition, $L(G) = \{w \in T^* \mid$ there is a derivation $u_1, u_2, \ldots, u_n$ in which $u_1 = S$ and $u_n = w\}$. So $L(G) = \{$the president, the mother of the president, the mother of the mother of the president, ...$\}$. Because $L(G) = A$, A is context-free.

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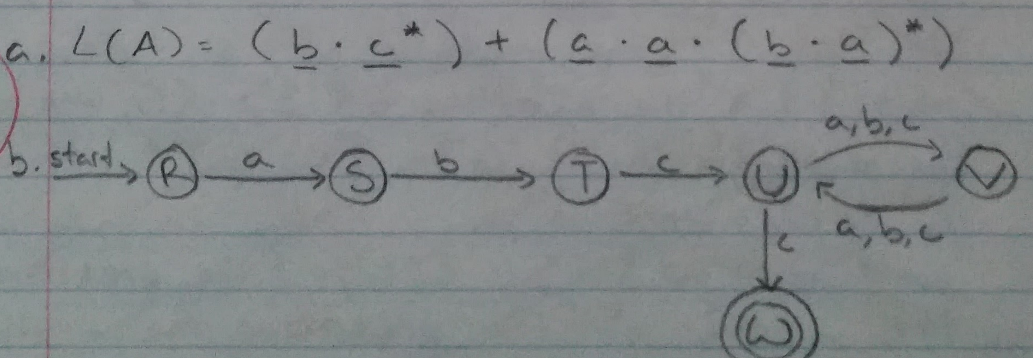
b. Natural languages are not adequately modeled by context-free grammars because CFGs cannot model cross-serial dependencies, which are present in some natural languages like Swiss German. Cross serial dependencies occur in a sentence where the object of the verb is the subject of another clause. The dependencies describe which subjects and verbs are constituents. In Swiss German the subjects and the verbs have the same order, but first all of the subjects are listed and then all the verbs (as opposed to a selfembedding or adjacent order). This causes the dependencies to cross. CFGs cannot model this because the dependencies require a memory to keep track of the subject/verb order, something that can't be implemented without context. As such, CFGs cannot adequately model natural language.

6/6

a. $L(A) = (\underline{b} \cdot \underline{c}^*) + (\underline{a} \cdot \underline{a} \cdot (\underline{b} \cdot \underline{a})^*)$

b.



a. The function $\mathcal{L}$ is not one-to-one because for every distinct $x, x' \in RE(A)$, it is not the case that $\mathcal{L}(x) \neq \mathcal{L}(x')$. For example, if $A = \{a, b\}$, $(\underline{a} + \underline{b})$ and $(\underline{b} + \underline{a})$ are two distinct elements of $RE(A)$. However, $\mathcal{L}(\underline{a} + \underline{b}) = \{a, b\}$ and $\mathcal{L}(\underline{b} + \underline{a}) = \{b, a\}$ and $\{a, b\} = \{b, a\}$. Therefore $\mathcal{L}$ is not one-to-one. $\{a^n b^n \mid n > 0\} \in P(A^*)$ is not regular.

X ~~$\mathcal{L}$ is onto~~ because the range of $\mathcal{L}$ is the same as the $\mathcal{P}(A^*)$. This is because the range of regular expressions that can be inputted into $\mathcal{L}$ is recursively infinite (if $s, t \in RE(A)$, then $(s + t)$, $(s \cdot t)$, and $s^* \in RE(A)$) and can thus denote any set of finite sequences of elements of A ($\mathcal{P}(A^*)$).

(Each finite sequence, which can be represented with some regular expression, can be unioned (+) with any number of other regular expressions representing finite sequences to form a set.)

b. $\mathcal{L}(RE(A)) = \{\mathcal{L}(r) \mid r \in RE(A)\}$

Show that $\bigcup \mathcal{L}(RE(A)) = A^*$

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Proof: To show that $\bigcup \mathcal{L}(RE(A)) = A^*$, show that ① $\bigcup \mathcal{L}(RE(A)) \subseteq A^*$ and ② $A^* \subseteq \bigcup \mathcal{L}(RE(A))$

① $\bigcup \mathcal{L}(RE(A))$ consists of all x such that for some $B \in \mathcal{L}(RE(A))$, $x \in B$. By definition, every $B = \mathcal{L}(r)$ such that $r \in RE(A)$. The function $\mathcal{L}$ is defined to be from RE into $\mathcal{P}(A^*)$, so $\mathcal{L}(r)$, where $r \in RE(A)$, will be some element of $\mathcal{P}(A^*)$. By definition of a powerset, $\mathcal{L}(r)$ then is a subset of A^*. Since $B = \mathcal{L}(r)$, $\bigcup \mathcal{L}(RE(A))$ then consists of all x such that $x \in$ some subset of A^*. Therefore every element of $\bigcup \mathcal{L}(RE(A))$ is an element of

A^*, so $\bigcup \mathcal{L}(RE(A)) \subseteq A^*$.

(2) As shown above, $\bigcup \mathcal{L}(RE(A))$ consists of all x such that $x \in$ some subset of A^*. Since any set is a subset of itself, $A^* \subseteq A^*$, so $\bigcup \mathcal{L}(RE(A))$ contains all $x \in A^*$. Therefore every element of A^* is an element of $\bigcup \mathcal{L}(RE(A))$, so $A^* \subseteq \bigcup \mathcal{L}(RE(A))$.

Because $\bigcup \mathcal{L}(RE(A)) \subseteq A^*$ and $A^* \subseteq \bigcup \mathcal{L}(RE(A))$, $\bigcup \mathcal{L}(RE(A)) = A^*$. $\square$