

Midterm Exam

Last Name: _____

First Name: _____

Student ID: _____

Signature: _____

Instructions: Do not open this exam until instructed to do so. You will have 1 hour and 50 minutes to complete the exam. Please print your name and student ID number above. You may not use books, notes, or any other material to help you. You will need a calculator for some problems. Please make sure **your phone is silenced** and stowed where you cannot see it. You may use any available space on the exam for scratch work. If you need more scratch paper, please ask one of the proctors.

Please do not write below this line.

Problem	Max	Score
1	10	
2	8	
3	10	
4	12	
5	8	
6	10	
Total	58	

1. (10 pts)

(a) (5 pts) Sketch a graph that does NOT represent a function and briefly explain why it is not a function.

(b) (5 pts) Why is it important that vector fields are functions? In other words, what could happen if they weren't?

2. (8 pts) Raccoons inhabit the UCLA campus. Write a differential equation for the number of raccoons, R , using the following assumptions.

- Every year, m raccoons come from the Santa Monica Mountains and join the UCLA population.
- The raccoon per-capita birth rate is b .
- The raccoon per-capita death rate is proportional to the ratio of the raccoon population to the number of trash cans, c , with proportionality constant k .

3. (10 pts) Koalas (K) feed exclusively on eucalyptus (E). Use the following assumptions to write a differential equation model of the food chain.
- Eucalyptus grows at a per-biomass (like per-capita) rate of 0.01.
 - Due to shading, the per-biomass rate at which eucalyptus dies is proportional to the amount of eucalyptus with a proportionality constant of 0.02.
 - Koalas consume eucalyptus at a per-capita rate proportional to the amount of eucalyptus available, with a proportionality constant of 0.05.
 - The koala birth rate is proportional to the amount of eucalyptus they consume, with a proportionality constant of 0.15.
 - Koalas die at a per-capita rate of 0.03.

4. (12 pts) A rabbit population is modeled by the differential equation

$$X' = 0.1X\left(1 - \frac{X}{100}\right)$$

(a) (8 pts) Suppose the current population is 120. Use Euler's method with a step size of 0.1 years to estimate the population size after 0.2 years.

(b) (4 pts) Sketch the vector field for this model.

5. (8 pts) The rate of change of acceleration is called *jerk*. A smartphone has a built-in accelerometer that measures acceleration so frequently that the record may be considered continuous. Describe how you could use data from the accelerometer to find the jerk of a person's movement at a particular moment, t_0 .

6. (10 pts)

(a) (5 pts) Find the linear approximation to the function $f(x) = 5x^3 + 10x^2 - x + 9$ at $x = 2$.

(b) (5 pts) Use this linear approximation to estimate $f(2.05)$. (You must use the linear approximation to get credit for this problem.)