

## ENG 111 Winter 2017 Final Exam – Solutions

1. (6 points) EB FARNUM Inc. has the following information on its new project.

Per Unit Variable Cost: \$4

Financial Break-Even Price (per unit): \$9.26

If the variable cost were to go up by 10%, what is the percentage increase needed in price for EB FARNUM Inc. to still financially break even?

*Financial break-even condition:*

$$(Price * Q - VC * Q - FC - Dep) * (1 - t) - (EAC - Dep) = 0$$

$$\text{Solve for } P \text{ to get Financial Break-Even Price} = \frac{(EAC - Dep)}{(1 - t)} + VC + \frac{FC}{Q} + \frac{Dep}{Q}$$

*Financial break-even price has a derivative of 1 with respect to variable cost.*

*If VC goes up by 10%, it goes up by \$0.4. Therefore, financial break-even price would go up by exactly the same amount to become \$9.66.*

*This is a  $(9.66 - 9.26) / 9.26 = 4.3\%$  increase.*

2. (6 points) A project will bring money for 4 years after its initial investment of \$5,000. In each of the first 2 years cash flow will be \$2,000. If the payback period is 3 years and discounted payback period is 3.5 years, what is the cash flow in the last year given APR is 10%? (Hint: Discounted payback period of a \$500 investment that brings \$1,100 in one year is 6 months if  $r=10\%$ )

$$3^{\text{rd}} \text{ year cash flow} = 5,000 - 2,000 - 2,000 = 1,000$$

*Let X be the cash flow in the last year (4<sup>th</sup> year). Then,*

$$2,000/1.1 + 2,000/1.1^2 + 1,000/1.1^3 + (X/1.1^4)/2 = 5,000$$

$$X = 2,277$$

3. (8 points) A company has a Net Income of \$95K, \$18K dividends and \$1M sales. Total assets are worth \$2M, one third of which is financed by debt (two thirds by equity). What is the growth rate of earnings?

$$g = b * ROE$$

$$ROE = PM * TAT * EM = 95K/\$1M * \$1M/\$2M * \$2M/(2/3 * \$2M) = 0.071$$

$$b = 1 - 18K/95K = 0.811$$

$$g = 0.071 * 0.811 = 0.058$$

4. (4 points) Jimmy McGill Inc. just distributed \$4 per share as dividends. Going forward, dividends are expected to grow by 5% each year. The market rate is 10%.

The following plan is going to be voted in the next board meeting of Jimmy McGill: Retain next year's dividend, distribute \$4 the following year and thereafter with an expected growth rate of 6%. As a stock holder of the company, would you vote yes or no for this plan? (Hint: Your objective is to have the highest price for the stock)

$$P_t = [D_{t+1}] / (R - g)$$

*Graders: As long as this formula is recognized and applied, most points should be given. For example for the NO vote, if the student calculated  $4/(0.1-0.05)$  instead of what we have below, no major points should be taken off.*

*Vote NO would result in:  $(1.05)4/(0.1-0.05) = \$84$  as the price of the stock today.*

*Vote YES would result in:  $[4/(0.1-0.06)]/(1.1^2) = \$82.64$  as the price of the stock today.*

**Vote NO.**

5. (3 points) You have observed the following returns on Corporation X's stock over the past five years: 34%, 16%, 19%, -21%, 8%. Suppose the average Treasury Bill rate was 1.2%.

a. What was your holding period return?

$$(1.34 * 1.16 * 1.19 * 0.79 * 1.08) - 1 = 57.82\%$$

b. What was the arithmetic average return on stock X over this five year period?

$$(34\% + 16\% + 19\% - 21\% + 8\%) / 5 = 11.2\%$$

d. What was the average risk premium on Corporation X's stock?

*The risk premium is the difference between the average return of X and the risk free rate, that is, the Treasury Bill rate.*

*Then, the real risk premium of Corporation X stock is  $11.2\% - 1.2\% = 10\%$*

6. (9 points) Your company is considering to undertake a new project that will last for 4 years. Initial investment is \$16K, that will depreciate to zero straight line over the life of the project with a market value of \$X that can be obtained in year 4. Tax rate is 34%. The following table gives the remaining costs and benefits of the project

	<b>Year 1</b>	<b>Year 2</b>	<b>Year 3</b>	<b>Year 4</b>
Sales	8,500	9,000	9,500	7,000
Costs	2,400	2,500	2,700	2,200
Increase in NWC	250	300	200	-750

What should be X for the NPV of the project to be positive if  $r=12\%$ ?

	<i>Year 1</i>	<i>Year 2</i>	<i>Year 3</i>	<i>Year 4</i>
<i>Sales</i>	8,500	9,000	9,500	7,000
<i>Costs</i>	2,400	2,500	2,700	2,200
<i>Increase in NWC</i>	250	300	200	-750
<i>Operating Cash Flow</i>	5,136	5,350	5,648	5,278

$$\text{Salvage Value} = X - t(X - \text{book value}) = X(1 - t) = 0.66X$$

$$\text{NPV} = -16,000 + 5,136/1.12 + 5,350/1.12^2 + 5,648/1.12^3 + 5,278/1.12^4 = 225$$

*Salvage Value does not have to be positive for the NPV to be positive. Therefore, the market price X, could be anything above zero.*

*Graders: Note that negative price does not make sense. Take off 1point!*

7. (8 points) You buy a 20-year Treasury Bond for a \$1,000, with 7% coupon rate and \$1,000 face value today. At the same time, your friend Alice buys a 2-year Treasury Bond for \$1,000 with 7.5% coupon rate and \$1,000 face value. Assume both Treasury bonds make coupon payments annually. Both you and Alice would like to sell your bonds, one year from today, right after you made your first coupon collections, when the market rate is 8%.

a. What is the return you got from your investment? What is Alice's return?

*Your return:*

*Price of your bond at 8% one year after the bond is issued:*

*Bond value of a 20-year bond with a coupon rate of 7%, and a face value of \$1,000 and with 19 years left to maturity is*

$$1,000/(1.08)^{19} + (70/0.08) * (1 - (1/(1+0.08)^{19})) = 903.96 \text{ and your return is } (70 + 903.96)/1000 - 1 = -2.6\% \text{ (4 points)}$$

*Alice's return:*

*Price of Alice's bond at 8% one year after the bond is issued:*

*Bond value of a 2-year bond with a coupon rate of 7.5%, and a face value of \$1,000 and with 1 year left to maturity is*

$$1,000/(1.08) + (75/(1+0.08)) = 995.37 \text{ and Alice's return is } (75 + 995.37)/1000 = 7.04\% \text{ (4 points)}$$

b. Your mutual friend, David, does not make the calculations and predicts that Alice got a higher return since Alice had the higher coupon rate. Is David correct? Why or why not?

*As the market rate becomes higher than the coupon rate, the price of the bond falls. It falls more for the longer term bonds. Therefore, Alice got a higher return not because she had a higher coupon but because she had the shorter term bond.*

8. (8 points) You invest \$X at 10% annually and withdraw half of the interest that accumulates during that year at the end of each year. How long would it take for your money to double?

*Your investment grows at 5% rather than 10%. This can be seen by writing the interest that accumulates every year:  $X + Xr/2 + (X + Xr/2)r/2 + (X + (X + Xr/2)/2)r/2 \dots$*

$$X(1+5\%)^n = 2X$$

$$n = 14.21$$

9. (10 points) Assume stocks A and B have the following characteristics:

Stock	Expected Return(%)	Standard Deviation(%)
A	11	20
B	15	34

The correlation between A and B is 0.3.

- a. A risk averse investor would like to invest some part of her money on a risk free asset which is expected to return 3% and the remaining on the minimum variance portfolio that is formed by A and B. What percentage of the portfolio is invested on stock A if, overall, her risk level (standard deviation) is 8%?

*Denote the overall portfolio formed by A, B, and risk free asset by P.*

*Denote the minimum variance portfolio that is formed by A and B by M.*

$$\sigma_M^2 = w_A^2 \sigma_A^2 + (1-w_A)^2 \sigma_B^2 + 2 w_A \sigma_A (1-w_A) \sigma_B \rho_{AB}$$

$$\sigma_M^2 = w_A^2 (0.2)^2 + (1-w_A)^2 (0.34)^2 + 2 w_A 0.20 (1-w_A) 0.34 * 0.3$$

$$\sigma_M^2 = w_A^2 0.04 + (1-w_A)^2 0.1156 + 0.0408 w_A (1-w_A)$$

*Minimize with respect to  $w_A$  by taking the first derivative and equating it to zero:*

$$0.08 w_A - 2(1-w_A) 0.1156 + 0.0408 - 2 * 0.0408 w_A = 0,$$

$$w_A = 0.8293$$

$$\text{Then, } \sigma_M^2 = 0.037$$

$\sigma_P^2 = w_M^2 \sigma_M^2$  (since the risk free asset has zero variance and zero correlation with the minimum variance portfolio formed above)

$$0.08^2 = 0.037 w_M^2$$

$$w_M = 0.4159$$

*Overall weight of stock A is  $0.4159 * 0.8293 = 0.35$*

- b. Another investor, who is not so risk averse, would like to invest on the minimum variance portfolio as well as the risk free asset and obtain an expected return of 19%. What percentage of his portfolio is going to be coming from borrowed money?

*Expected return of minimum variance portfolio is*

$$0.8293 * 0.11 + (1 - 0.8293) * 0.15 = 0.1067$$

$$19\% = 3\% w_{\text{riskfree}} + 11.68\% (1 - w_{\text{riskfree}})$$

$$w_{\text{riskfree}} = -0.8433$$

*If the investor has \$100, she will invest a total of \$184.33 where \$84.33 of her portfolio ( $\$84.33 / \$184.33 = 46\%$ ) is coming from borrowed money.*

10. (6 points) You invested half of your money on stock Y and the other half on stock Z. Your portfolio has a beta of 1.4. Stock Y has a beta of 1.2 and an expected return of 8%. If the risk-free rate is 3%, what is the expected return of your portfolio? (Hint: Portfolio's beta is the weighted average of individual betas.)

$$\text{Portfolio beta} = 0.5 \text{ beta of Y} + 0.5 \text{ beta of Z}$$

$$1.4 = 0.5 * 1.2 + 0.5 * \text{beta of Z}$$

$$\text{beta of Z} = 1.6$$

$$\text{Expected return of Y} = \text{risk-free rate} + \text{beta of Y} * (\text{market's expected rate} - \text{risk-free rate})$$

$$0.08 = 0.03 + 1.2 * (\text{market's expected rate} - \text{risk-free rate})$$

$$(\text{market's expected rate} - \text{risk-free rate}) = 0.0417$$

$$\text{Expected return of Z} = \text{risk-free rate} + \text{beta of Z} * (\text{market's expected rate} - \text{risk-free rate})$$

$$\text{Expected return of Z} = 0.03 + 1.6 * 0.0417$$

$$\text{Expected return of Z} = 9.7\%$$

$$\text{Expected return on portfolio} = 0.5 * 8\% + 0.5 * 9.7\% = \mathbf{8.85\%}$$

11. (8 points) You purchased a 30-year bond with a \$1,000 face value and a 10% coupon rate at a price of \$1,000 exactly two years ago at its issue date and collected your second coupon today. The market rate is currently 13%. You would like to sell your bond by next year. That is, you either have to sell your bond today or a year from today (for simplicity, assume you can sell next year immediately after you collect your 3<sup>rd</sup> coupon payment and assume you cannot sell the bond between now and then).

a) If you expect the market rate to stay at 13% in the future, would you sell your bond now or next year?

In order to compare the two options we need to be careful about comparing values that belong to the same period: Compare the price of the bond today versus price of the bond plus the coupon to be collected next year DISCOUNTED TO TODAY!!!

$$\text{Sell now: Price of the bond} = 1,000/1.13^{28} + (100/0.13) * (1-1/1.13^{28}) = \$776.76$$

Sell next year:

$$\text{Price of the bond next year} = 1000/1.13^{27} + (100/0.13) * (1-(1/1.13^{27})) = \$777.74$$

$$\text{Value today} = (100 + 777.74)/1.13 = \$776.76$$

**You are indifferent between selling now and selling next year!**

b) What if you expect the rates to go down next year? In particular, if you know that the market rate will be 7% at the time you try to sell next year... would you sell now or next year?

*Sell now: Price of the bond =  $1,000/1.13^{28} + (100/0.13) * (1-1/1.13^{28}) = \$776.76$*

*Sell next year:*

*Price of the bond next year =  $1000/1.07^{27} + (100/0.07) * (1-(1/1.07^{27})) = \$1,359.60$*

*Value today =  $(100+1,359.60)/1.13 = \$1,203.19$  (note that the opportunity cost of money from today to next year is 13%, that's why we need to discount the price of bond we would obtain and coupon we would obtain next year by 13% not by 7%)*

***You would sell next year!** (Note that this is a good example pointing out why people would want to have longer term bonds if they expect the rates to go down in future!)*

**12. (6 points)**

a) What would happen to short term bond yields if a large number of investors expect a recession in future?

*They would invest in the long term bonds driving the price up and yields down for the long term bonds. Conversely, short term bond prices would go down and **yields would go up.***

b) What would happen to bond yields if a large number of investors switch their investment preferences and prefer to invest on the stock market rather than the bond market?

*When the demand for any product goes down, the price is expected to go down. If demand for bonds go down, the price of bonds would go down and the yields of bonds would go up.*

c) What would happen to bond yields if a large number of investors expect inflation to be high in the future?

*Bond yields would be higher due to inflation premium.*

13. (12 points)

BOND DESCRIPTION: Today, you are considering purchasing a 3-year discount bond. There are no coupon payments, only the face value of \$1,000 to be paid 3 years from today. Bond is selling at a price \$X today.

a) At the current price, bond has a promised yield of 12%. Your alternative is 10% APR. The probability of default is p and in case of default bond will pay \$200 instead of \$1,000. What is the expected default rate?

$$\text{Bond Price} = 1,000/1.12^3 = \$711.78$$

$$\$711.78 = p(200/1.1) + (1-p)(1000/1.1) \text{ which implies } p = 27\%$$

*Alternatively, you could think the above equation in terms of future values. If I invest \$711.78 today, in 3 years, its future value is  $\$711.78 \times (1.1)^3$  to me. Then, the expected future value that is offered by the bond should be the same:*

$$\$711.78 \times (1.1)^3 = p200 + (1-p)1000 \text{ which implies } p = 27\% \text{ as before!}$$

b) Suppose BOND DESCRIPTION stays the same but you get insider information and learn that the bond would not default, what is the NPV of your investment?

$$\text{NPV} = -711.78 + 1000/1.1^3 = \$39.53$$

c) Suppose BOND DESCRIPTION stays the same but you get insider information and learn that the default rate is 80%. Would you be willing to buy the bond at a price of \$X today? If not, what would be the price that you would pay for this bond today?

*You would not be willing to purchase at the current price. The value of the bond would be  $(0.8 \times 200 + 0.2 \times 1000)/1.1^3 = \$270.47$  and that would be the maximum you would pay.*



14. (6 points) A stock has a beta of 1.3 and an expected return of 8%. If the risk free rate is 3% what would be the expected return of a stock with a beta of 1.8 ?

$$\text{CAPM: } E(r_i) = r_F + \beta_i(r_M - r_F)$$

$$0.08 = 0.03 + 1.3(r_M - r_F)$$

$$(r_M - r_F) = 0.038$$

$$0.03 + 1.8 * 0.038 = 9.9\%$$