

ENG 111 , WINTER 2015 , FINAL SOLUTIONS: Multiple Choice Questions (3 points each)

1. Managers are encouraged to act in shareholders' interest by:

- a) Shareholder election of a board of directors who select management.
- b) The threat of a takeover by another firm.
- c) Compensation contracts that tie compensation to corporate success.
- d) Only B and C.
- e) A, B, and C.

2. Consider two projects with the following cash flow:

Year	Project A	Project B
1	1,200	4,800
2	2,400	3,600
3	3,600	2,400
4	4,800	1,200

Which of the following is true concerning these two projects?

- a) Both projects have the same future value at the end of 4<sup>th</sup> year regardless of the interest rate.
- b) Given a positive interest rate, the future value of Project B is higher than that of Project A.
- c) If the payback period method rejects Project A, it would also reject Project B.
- d) If the payback period method accepts Project B, it would also accept Project A.
- e) None of the above.

3. Your company will undertake a new project. When evaluating the NPV of the project, all of the following must be included in calculations except,

- a) Office building to be used for the project that the company already owns.
- b) The projected decrease in revenue from the company's existing product(s) due to the introduction of the new project.
- c) The expenses that are already paid to a market research company for testing the viability of the new project.
- d) An equipment that must be purchased to use in the project.
- e) None of the above.

4. Which of the following is not correct?

- a) If the correlation between two assets is -1, then, there exists a portfolio of these two assets with a standard deviation of 0.
- b) If the return of asset A is twice as sensitive to market changes as asset B, then investors would demand twice as much risk premium (extra return over the risk free rate) from A.
- c) In a big and well-diversified portfolio, idiosyncratic risk is minimized.
- d) Increasing the number of assets in a portfolio does not necessarily reduce the systematic risk.
- e) If a portfolio consists of two assets with a positive correlation between them, the standard deviation of the portfolio has to be higher than individual asset standard deviations.

5. "Proxy Fight" which we discussed within the context of the current events (General Motors) usually arises as a result of

- a) the dissatisfaction of the board members with the management.
- b) management's dissatisfaction with the board of directors.
- c) a group of shareholders' dissatisfaction with the current management.
- d) management's fight with the competitors to increase market share.
- e) none of the above.

6. Everything else being equal,

- a) a zero coupon bond has lower interest rate risk compared to a positive coupon bond.
- b) a zero coupon bond has higher reinvestment risk compared to a positive coupon bond.
- c) a bond with a longer maturity has higher reinvestment risk compared to a bond with a shorter maturity.
- d) a bond with a longer maturity has lower reinvestment risk compared to a bond with a shorter maturity.
- e) none of the above.

7. At the financial break-even point of a project,

- a) payback period must be less than the length of the project.
- b) your return on the project is exactly the same as your best alternative.
- c) you cover all of your costs except your opportunity cost.
- d) A and B are correct.
- e) A, B, and C are correct.

8. Given that an investor has the option of investing on a highly diversified portfolio as well as the risk-free asset:

- a) The optimal composition of the risky assets depends on investor's risk level.
- b) Desired risk level of the investor will help determine the composition of risky assets.
- c) The percentage of the portfolio that is invested on the risk-free asset depends on the risk level of the investor.
- d) It is always best to invest 50% of the portfolio on the risk-free asset.
- e) None of the above

9. An analysis technique calculating the effect of change in one variable on the net present value is called \_\_\_\_\_ analysis.

- a) forecasting
- b) scenario
- c) sensitivity
- d) simulation
- e) break-even

10. If the current yield is greater than the coupon rate,

- a) market rate is greater than Yield to Maturity.
- b) market rate is less than Yield to Maturity.
- c) coupon rate is higher than the market rate.
- d) coupon rate is less than the market rate.
- e) none of the above.

**Numerical/Concept Questions:**

11. (8 points) Your firm is considering a project with a five-year life and an initial cost of \$120,000. The firm expects to sell 2,100 units per year at a price of \$20 per unit. Market rate is 12%. The firm will have the option to abandon this project after three years at which time it expects it could sell the project for \$50,000. At what level of sales should the firm be willing to abandon this project?

*There are several ways of solving this problem, all leading to the same conclusion:*

- I. *After the 3<sup>rd</sup> year production ends, the firm can continue with the production with Q units per year and have 20 x Q as cash flow per year. Alternatively, firm can stop and get 50,000 at the end of the 3<sup>rd</sup> year. To compare these two options we need to discount the cash flow that is going to be coming in years 4 and 5 to*

$$\text{year 3: } \$50,000 = (Q \times \$20) \times \frac{1 - [1/(1 + .12)^2]}{.12}; \$50,000 = 33.801Q; Q = 1,479 \text{ units}$$

- II. *We can consider the project as a whole and write down the discounted cash flows of both options as of time 0 and compare assuming that for the first three years, production is going to be 2,100, hence cash flow 2,100\*20=42,000.*

$$-120,000 + \frac{42,000}{1.12} + \frac{42,000}{1.12^2} + \frac{42,000}{1.12^3} + \frac{20Q}{1.12^4} + \frac{20Q}{1.12^5} <$$

$$-120,000 + \frac{42,000}{1.12} + \frac{42,000}{1.12^2} + \frac{42,000}{1.12^3} + \frac{50,000}{1.12^3} \text{ which reduces to}$$

$$\frac{20Q}{1.12} + \frac{20Q}{1.12^2} < 50,000, \text{ then gives, } Q < 1,479.25$$

- III. *Since the decision is based on what will happen after the third year, the quantities for the previous years are irrelevant.*

$$\text{Then, } \frac{20Q}{1.12^4} + \frac{20Q}{1.12^5} < \frac{50,000}{1.12^3}, \text{ that is,}$$

$$\frac{20Q}{1.12} + \frac{20Q}{1.12^2} < 50,000, \text{ which gives, } Q < 1,479.25.$$

12. (5 points) Flattening or inverted yield curve is considered as an indication of economic downturn. What would make the yield curve invert?

*Expectations of the consumers and businesses affect the future health of the economy. If consumers are expecting a downturn, they will not purchase as much now to save. If businesses expect a downturn, they will not invest and hire. They will cut back production. This will create an*

*expectation of low returns in the future. If enough investors feel this way, then, they will prefer safe long term investment options. If this takes a form of selling short term bonds and buying long term bonds, then the price of short term bonds will decline to give us high short term yields while the price of long term bonds will increase giving us low long term yields. This yield difference will make the yield curve flatten or even invert in some cases.*

**13.** You bought a bond with the following specifications on March 16<sup>th</sup> of 2014.

Face Value: \$1,000

Time to Maturity: 2 years

Coupon Rate: 10%

On the day you bought the bond, the return you could obtain elsewhere was 10%. Today, you can obtain 12% in the market and you just collected your \$100 coupon payment.

a) **(3 points)** Should you sell your bond today? Why or why not?

*The value of the bond today is  $1,100/1.12 = \$982.14$*

*If I do not sell the bond I will get \$1,100 in one year's time.*

*If I can sell the bond at a price of \$X where  $X > \$982.14$ , I should sell it because,  $X(1.12) > \$1,100$ .*

*If I can sell the bond at a lower price than \$982.14, I should not sell it.*

*If I can sell the bond at exactly \$982.14, which should be the market price of this bond when it is correctly priced, then I am indifferent between selling and not selling. Both options will give me \$1,100 in one year.*

b) **(3 points)** If you sell your bond and reinvest the money at the market rate, what would be the geometric average return that you would have obtained over the course of two years?

*Assuming that I sold the bond at \$982.14, my return in first year is  $(982.14 + 100 - 1,000)/1,000 = \%8.214$ , return in second year is 12%.*

*Geometric average return is  $[(1.08214)(1.12)]^{1/2} - 1 = \%10.09$   
Thus far is enough to get full points.*

*If I could sell the bond at a higher price than \$982.14, then my geometric average return would be higher.*

*\*\* Note that if you do not sell the bond it looks like you are going to be obtaining only 10% average over the course of two years. But remember that, when you collect your \$100 at the end of the first year, you can invest it at 12% which makes your overall average comparable to 10.09% above!*

**14. (6 points)** Bruins Inc. is a startup. It is estimated that the company will not be paying any dividends for the coming 6 years because it needs to use its earnings to fuel growth. The company is expected to pay dividends of \$3.6 a share 7 years from today and will increase the dividends at a 4% per year thereafter. If the rate that is expected from a company with equal risk is 12%, if you purchase one Bruins stock today to sell it one year from today, what would be your expected return?

*Using a constant growth stock pricing, Bruins stock price at  $t=6$ ,  $3.6/(.12-.04) = \$45$   
Price at  $t=0$  is  $45/1.12^6$ , price at  $t=1$  is  $45/1.12^5$ , which gives a return of 12%, as expected, it is the same as your alternative investment.*

**15. (7 points)** The Zengels Brewing Company recently installed a new bottling machine. The machine's initial cost is \$2,000 that will be depreciated on a straight-line basis to a zero-salvage in 5 years. The machine's fixed cost per year is \$1,800 and its variable cost is \$0.50 per unit. The selling price per unit is \$P. Zengels' tax rate is 34% and it uses a 16% discount rate. If Zengels needs to produce 320 more units to be able to financially break even compared to the accounting break-even sales level, what is P?

*Accounting break-even is:*

$$(\$1,800 + \$400)(1 - 0.34)/(\$P - \$0.5)(1 - .34) = X + 320 \text{ units}$$

*Financial break-even is:*

$$EAC = \$2,000 * 0.16 / (1 - 1/(1.16^5)) = \$2,000 / 3.2743 = \$610.81$$

$$\begin{aligned} & (EAC + \text{Fixed Costs} * (1-t) - t * \text{Depr.}) / (\text{Sales Price} - \text{Var. Cost}) * (1-t) \\ & = [\$610.81 + \$1,800(1 - .34) - \$400(.34)] / (\$P - \$0.5)(1 - .34) = X \text{ units} \end{aligned}$$

*Solving these two equations give P approximately \$1.5.*

**16. (6 points)** You invested on a portfolio with expected return of 18% and a standard deviation of 41%. Assuming that the returns follow a normal distribution, with what probability your money will at least double next year?

$$(100 - 18) / 41 = 2,$$

*The rate being within two standard deviations is 95%. The rate being higher than %100 happens with probability  $(1 - .95) / 2 = 0.025$  (or 2.5%).*

**17.** You have three possible investment opportunities with cash flows given in the following table. Your market rate is 20%.

Year	A	B	C
0	-500	-50	-400
1	100	100	200
2	200	100	300
3	500	200	50

a) **(2 points)** Assume that projects A, B, and C are independent and you do not have any budget constraints. Which project(s) would you choose using the Profitability Index?

$$PI_A = (100/1.2 + 200/1.2^2 + 500/1.2^3) / 500 = 1.023$$

$$PI_B = (100/1.2 + 100/1.2^2 + 200/1.2^3) / 50 = 5.37$$

$$PI_C = (200/1.2 + 300/1.2^2 + 50/1.2^3) / 400 = 1.009$$

*All are greater than 1, indicating all positive NPV. Pick all.*

- b) i) **(2 points)** If the projects are independent, and you have a budget constraint of 500, which project(s) would you invest on? (you can invest on a project only once).

*You have two options: Option I is to invest solely on A.*

*Option II is to invest on B and C and the market.*

*You can evaluate these two options in several ways, all of which are acceptable.*

*You can calculate the NPV and compare. You can calculate the future value and compare.*

*You can also do an incremental PI evaluation.*

*The following calculation follows obtaining the future value at year 3. Comparison can be made by carrying the cash flow to a different year as well:*

*Option I:*

$$-500*1.2^3+100*1.2^2+200*1.2+500= 20$$

*Option II:*

$$-450*1.2^3+300*1.2^2+400*1.2+250= 384.4$$

*Plus, the remaining 50 can be invested at the market rate which will give  $50*1.2^3 = 86.4$*

*(If the student did not consider the remaining 50, take off 0.25 points)*

*Pick option II (Invest on B and C).*

- ii) **(2 points)** If instead, you invest your 500 at a fixed rate of  $r\%$  per year and have the same amount at the end of the 3<sup>rd</sup> year as in part (i), what is  $r$ ?

$$-450*1.2^3+300*1.2^2+400*1.2+250+50*1.2^3 = 384.4+ 86.4 = 470.8$$

$$-500*1.2^3+500*(1+r)^3 = 470.8$$

*Solve for  $r$ ,  $r = 38.72\%$*

- c) **(2 points)** If, instead, the projects were to be mutually exclusive, which project would you choose, using the Profitability Index?

$$P_{(C-B)} = (100/1.2+200/1.2^2-150/1.2^3)/350 = 0.39, B \text{ is better than } C$$

$$P_{(A-B)} = (0/1.2+100/1.2^2+300/1.2^3)/450 = 0.54, B \text{ is better than } A$$

*Choose B.*

**18.** Assume that the market rate is 13%. Linkedout Inc. has \$6.25 EPS which is expected to stay the same if the firm makes no new investment and returns the earnings as dividends to the shareholders. The new CEO would like to change the company policy and retain 20% of the earnings beginning three years from today. He expects to invest the retained earnings each year and earn 11% return perpetually beginning one year after each investment is made.

- a) **(2 points)** What is the price per share of Linkedout stock today if the company stays as cash cow and does not make any new investments?

*If the company does not make any new investments, the stock price will be the present value of the constant perpetual dividends. We need to find the stock price of the firm as a cash cow. In this case, all earnings are paid as dividends, so, applying the perpetuity equation, we get:*

$$P = \text{Dividend} / R$$

$$P = \$6.25 / 0.13$$

$$P = \$48.08$$

- b) **(3 points)** If the new CEO changes the policy as described above, what would be the price per share now?

*We can think of at least two ways of solving this problem. The first one is: The investment occurs every year in the growth opportunity, so the opportunity is a growing perpetuity. So, we first need to find the growth rate. The growth rate is:*

$$g = \text{Retention Ratio} \times \text{Return on Retained Earnings}$$

$$g = 0.20 \times 0.11$$

$$g = 0.022 \text{ or } 2.20\%$$

*Next, we need to calculate the NPV of the investment. During year 3, 20 percent of the earnings will be reinvested. Therefore, \$1.25 is invested ( $\$6.25 \times .20$ ). One year later, the shareholders receive an 11 percent return on the investment, or \$0.138 ( $\$1.25 \times .11$ ), in perpetuity. The perpetuity formula values that stream as of year 3. Since the investment opportunity will continue indefinitely and grows at 2.2 percent, apply the growing perpetuity formula to calculate the NPV of the investment as of year 2. Discount that value back two years to today.*

$$NPVGO = [(\text{Investment} + \text{Return} / R) / (R - g)] / (1 + R)^2$$

$$NPVGO = [(-\$1.25 + \$0.138 / .13) / (0.13 - 0.022)] / (1.13)^2$$

$$NPVGO = -\$1.39$$

*The value of the stock is the PV of the firm without making the investment plus the NPV of the investment, or:*

$$P = PV(\text{EPS}) + NPVGO$$

$$P = \$48.08 - 1.39$$

$$P = \$46.68$$

Alternative solution:

Cash Flow with the investment:

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
	6.25	6.25	0.8*6.25	6.25+6.25*0.20*0.11 =6.25 (1+g) 0.8	6.25 (1+g) <sup>2</sup> 0.8

The pattern of dividends after year 5 is apparent from the above table. For year 6, dividends will be  $6.25 (1+g)^3 0.8$ , and for year 7, they will be  $6.25 (1+g)^4 0.8$ . etc...

Price of the stock in year 2 with the investment =  $6.25*0.80/r-g = 5/.13-.022 = 46.29$

Price of stock in year 0 will be then:  $46.29/(1.13)^2 + 6.25/(1.13) + 6.25/1.13 = \$46.68$

- c) **(3 points)** Is 20% retention ratio the optimal ratio chosen by the new CEO given the above investment opportunity? If not, what should be the retention ratio?

*Zero percent! There is no retention ratio which would make the project profitable for the company. If the company retains more earnings, the growth rate of the earnings on the investment will increase, but the project will still not be profitable. Since the return of the project is less than the required return (market rate) on the company stock, the project is never worthwhile. In fact, the more the company retains and invests in the project, the less valuable the stock becomes.*

19. Consider two stocks, A and B, with the following characteristics:

Stock	Expected Return(%)	Standard Deviation(%)
A	9	22
B	15	45

The covariance between the returns of A and B is 0.001.

- a) **(3 points)** The expected return on the minimum variance portfolio that is formed by A and B equals to 10.14%. What is the weight of A in minimum variance portfolio?

Let  $E(.)$  denote the expected value,  $w_A$  denote the share of stock of A and  $w_B$  denote the share of stock B.

$$E(R_P) = w_A E(R_A) + w_B E(R_B)$$

$$E(R_P) = w_A E(R_A) + (1-w_A) E(R_B)$$

$$0.1014 = w_A (.09) + (1-w_A)(0.15)$$

$$w_A = .8096$$

- b) **(3 points)** What is the standard deviation of minimum variance portfolio?

$$\sigma_P^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \text{Cov}(A,B)$$

$$\sigma_P^2 = .8096^2 .22^2 + .1904^2 .45^2 + 2 *.8096 *.1904 *.001$$

$$\sigma_P^2 = .0394$$

The square root is the standard deviation: 19.84%



- c) **(3 points)** A risk averse investor would like to form a portfolio by investing on the minimum variance portfolio and the risk free asset. Overall, she does not want to have a risk level (standard deviation) higher than 10%. What is the maximum expected return she can get if the risk free rate is 3%?

*The standard deviation of the new portfolio is 10%. If the weight of risk-free asset is  $w_F$  and the weight of the min-var portfolio is  $w_M$  we have:*

$$(0.10)^2 = (w_F \sigma_F)^2 + (w_M \sigma_M)^2 + 2w_F w_M \text{Cov}(F, M)$$

*Since the standard deviation of the risk-free rate and the covariance between the risk-free rate and the minimum-variance portfolio rate are zero, we have:*

$$(0.10)^2 = (w_M \sigma_M)^2 = (w_M)^2 0.0394.$$

*Hence, the percentage invested on minimum-variance portfolio is 50.38%. Then the weight of the risk-free asset is  $1 - 0.5038 = 0.4962$ .*

$$\text{Maximum expected return} = .5038 * .10.14 + .4962 * .03 = 6.60\%$$

- 20. (7 points)** Portfolio K combines the risk-free asset and the market portfolio.

Portfolio K:

Expected Return: 9%

Standard Deviation: 13%.

Risk-free rate: 5%.

Expected Return of Market Portfolio: 12%

Assuming that the capital asset pricing model holds, what would be the expected return on a stock that has a standard deviation of 40% and correlation with the market portfolio of 0.45?

*First, we can calculate the standard deviation of the market portfolio using the Capital Market Line (CML). We know that the risk-free rate asset has a return of 5 percent and a standard deviation of zero and the portfolio has an expected return of 9 percent and a standard deviation of 13 percent. These two points must lie on the Capital Market Line. The slope of the Capital Market Line equals:*

*Slope<sub>CML</sub> = Increase in expected return / Increase in standard deviation*

$$\text{Slope}_{CML} = (.09 - .05) / (.13 - 0)$$

$$\text{Slope}_{CML} = .31$$

*According to the Capital Market Line:*

$$E(R_i) = R_f + \text{Slope}_{CML}(\sigma_i)$$

*Since we know the expected return on the market portfolio, the risk-free rate, and the slope of the Capital Market Line, we can solve for the standard deviation of the market portfolio which is given by:*

$$E(R_M) = R_f + \text{Slope}_{CML}(\sigma_M)$$

$$\begin{aligned} .12 &= .05 + (.31)(\sigma_M) \\ \sigma_M &= (.12 - .05) / .31 \\ \sigma_M &= .2275 \text{ or } 22.75\% \end{aligned}$$

Next, we can use the standard deviation of the market portfolio to solve for the beta of a security. Doing so, we find the beta of the security as:

$$\begin{aligned} \beta_I &= (\rho_{I,M})(\sigma_I) / \sigma_M \\ \beta_I &= (.45)(.40) / .2275 \\ \beta_I &= 0.79 \end{aligned}$$

Now we can use the beta of the security in the CAPM to find its expected return, which is:

$$\begin{aligned} E(R_I) &= R_f + \beta_I[E(R_M) - R_f] \\ E(R_I) &= 0.05 + 0.79(.12 - 0.05) \\ E(R_I) &= .1054 \text{ or } 10.54\% \end{aligned}$$

<b>Market Value Measures</b>	<p>Market Capitalization = Price per share * # Shares Outstanding  P/E Ratio = Price Per Share / Earnings Per Share  Market to Book Ratio = Market Value per Share / Book Value per Share  Enterprise Value = Market Capitalization + Market Value of Interest Bearing Debts – Cash  EV Multiple = EV/ EBIDTA</p>
<b>External Financing Formulas</b>	$EFN = \left( \frac{\text{Assets}}{\text{Sales}} \right) \times \Delta \text{Sales} - \frac{\text{Spon Liab}}{\text{Sales}} \times \Delta \text{Sales} - (PM \times \text{Projected Sales}) \times (1 - d)$ $\text{Internal Growth Rate} = \frac{ROA \times b}{1 - ROA \times b} \qquad \text{Sustainable Growth Rate} = \frac{ROE \times b}{1 - ROE \times b}$
<b>Present Value Formulas</b>	$FV = C_0 \times \left( 1 + \frac{r}{m} \right)^{m \times T} \qquad FV = C_0 e^{rT} \qquad PV = C / r \qquad PV = \frac{C}{r - g}$ $PV = \frac{C}{r} \left[ 1 - \frac{1}{(1+r)^T} \right] \qquad PV = \frac{C}{r - g} \left[ 1 - \left( \frac{1+g}{(1+r)} \right)^T \right]$
<b>Accounting Ratios</b>	<p>Current Ratio = Current Assets/ Current Liabilities  Quick Ratio = (Current Assets – Inventory) / Current Liabilities  Cash Ratio = Cash / Current Liabilities  Total Debt Ratio = (Total Assets – Total Equity ) / Total Assets  Debt/Equity = Total Debt / Total Equities  Equity Multiplier = Total Assets / Total Equity  Times Interest Earned = (Earnings Before Interest And Taxes) / Interest  Cash Coverage = (EBIT + Depreciation + Amortization) / Interest  Inventory Turnover = Cost of Goods Sold / Inventory  Days' Sales in Inventory = 365 / (Inventory Turnover)  Receivables Turnover = Sales / Accounts Receivable  Days' Sales in Receivables = 365 / Receivables Turnover  Total Asset Turnover = Sales / Total Assets  Profit Margin = Net Income / Sales  Return on Assets = Net Income / Total Assets  Return on Equity = Net Income / Total Equity  EBITDA Margin = EBITDA / Sales  Capital Intensity = Total Assets / Sales</p>
<b>Break Even Point</b>	<p>Accounting: (Fixed Costs+Depr.)/(Sales Price-Variable Cost)  Financial(Pres. Value): (EAC+Fixed Costs*(1-t) – t*Depr.) / (Sales Price-Var. Cost)*(1-t)</p>
<b>Bond Value</b>	$\text{Bond Value} = C \left[ \frac{1 - \frac{1}{(1+r)^T}}{r} \right] + \frac{F}{(1+r)^T}$

<b>Stock Valuation</b>	<p>Zero Growth: <math>P_0 = \frac{\text{Div}}{R}</math></p> <p>Constant Growth: <math>P_0 = \frac{\text{Div}_1}{R - g}</math></p> <p>Differential Growth: <math>P = \frac{C}{R - g_1} \left[ 1 - \frac{(1 + g_1)^T}{(1 + R)^T} \right] + \frac{\left( \frac{\text{Div}_{T+1}}{R - g_2} \right)}{(1 + R)^T}</math></p>
<b>Stock Returns</b>	<p>Holding Period Return: <math>HPR = (1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_T) - 1</math></p> <p>Arithmetic Average Return: <math>\bar{R} = \frac{(R_1 + \dots + R_T)}{T}</math></p> <p>Geometric Average Return: <math>\sqrt[T]{(1 + R_1)(1 + R_2) \dots (1 + R_T)} - 1</math></p>
<b>Sample Statistics</b>	<p><math>\bar{R} = \frac{(R_1 + \dots + R_T)}{T}</math>    <math>SD = \sqrt{VAR} = \sqrt{\frac{(R_1 - \bar{R})^2 + (R_2 - \bar{R})^2 + \dots + (R_T - \bar{R})^2}{T - 1}}</math></p> <p><math>Cov(A, B) = \sigma_{AB} = \frac{\sum_i (a_i - \bar{a})(b_i - \bar{b})}{T - 1}</math></p> <p><math>Corr(A, B) = \rho_{A, B} = \frac{\sigma_{A, B}}{\sigma_A \sigma_B}</math></p>
<b>General Case</b>	<p><math>E(A) = \sum_i p_i a_i</math>    <math>SD(A) = \sigma_A = \sqrt{\sum_i p_i (a_i - \bar{a})^2}</math></p> <p><math>Cov(A, B) = \sigma_{AB} = \sum_i p_i (a_i - \bar{a})(b_i - \bar{b})</math></p>
<b>Portfolio Analysis</b>	<p>Expected Return on Portfolio: <math>E(r_p) = x_A E(r_A) + x_B E(r_B)</math></p> <p>Variance of a portfolio: <math>\sigma^2 = x_A^2 \sigma_A^2 + 2x_A x_B \sigma_{AB} + x_B^2 \sigma_B^2</math></p> <p><math>\beta_i = \frac{Cov(R_i, R_M)}{\sigma^2(R_M)}</math></p>