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UCLA - ENG 110 Midterm

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1. (25 points) A company producing in a perfectly competitive industry has the following cost structure:

$$TFC = 20$$

$$TVC(Q) = 50Q^2$$

Assume that there are 100 identical companies operating in this industry.

- a) (10 points) What is the market price if the industry demand is given by $P = 2480 - 30Q$?

$$MC = 100\varphi_f \Rightarrow P = \varphi_f + \frac{20}{100} \Rightarrow \varphi_f = 100\varphi_f \Rightarrow \varphi_f = P$$

if demand = supply

$$2480 - 30\varphi_f = \varphi_f \Rightarrow 2480 = 31\varphi_f \Rightarrow \varphi_f^* = 80$$

$$P = 2480 - 30\varphi_f^* = 2480 - 30(80) = 2480 - 2400 = 80 \text{ (Ans)}$$

- b) (5 points) What is the profit obtained by each firm?

$$\pi(\varphi_f^*) = \text{M.R.} - \text{Total Cost}$$

$$(\varphi_f^*)^2 \cdot 80 = 80\varphi_f^* - \varphi_f^*/100 = 80(80) - 0.8$$

$$\pi(\varphi_f^*) = 80 \times 0.8 - (20 + 50 \times 8 \times 0.8) = 64 - 52 = 12 \text{ (Ans)}$$

- c) (10 points) How will this industry change in the long run? Will there be any entry? If so, let the total number of firms after the entry be x . Write the equation that will solve for x . DO NOT solve the equation as it involves higher order polynomials.

(For ease of calculation assume that any company that enters this industry will have the exact same scale and cost structure as others.)

Yes, there will be entry of firms until the point where each firm makes zero economic profit (in the long run).

If there are x firms in the market:

$$MC = 100\varphi_f \Rightarrow \varphi_f = \frac{P}{100} \Rightarrow \varphi_f = \frac{xP}{100} \Rightarrow P = \frac{100\varphi_f}{x}$$

$$\text{Demand = Supply} \Rightarrow \frac{100\varphi_f^*}{x} = 2480 - 30\varphi_f^* \Rightarrow \varphi_f^* = \frac{2480x}{100+30x}$$

$$\pi(\varphi_f^*) = 2480 - 30 \left[\frac{2480x}{100+30x} \right]; \varphi_f^* = \frac{\pi(\varphi_f^*)}{x} = \frac{2480x}{100+30x} - 30 = 0 \text{ in the long run with } x \text{ firms}$$

$$\therefore \pi(\varphi_f^*) = \left[2480 - 30 \left(\frac{2480x}{100+30x} \right) \right] \cdot \frac{2480x}{100+30x} - 30 = 50 \left[\frac{2480x}{100+30x} \right]^2 - 30 = 0$$

$$\therefore \left[\frac{2480x}{100+30x} \right]^2 - 30 = 50 \left[\frac{2480x}{100+30x} \right]^2 - 30 = 0 \text{ (Ans)}$$

2. (25 points) There are two firms, A and B, producing a homogenous product, X. Total quantity demanded (by the entire market) at each price is given in the following table for product X. Ignoring the production costs, and assuming that each firm simultaneously decides how many units to produce, what will be the price in equilibrium?

(Assume that each firm prefers to produce less when the revenue obtained is the same at different quantities.)

| Price | Total Quantity Demanded |
|-------|-------------------------|
| 1 | 132 |
| 2 | 120 |
| 3 | 108 |
| 4 | 96 |
| 5 | 84 |
| 6 | 72 |
| 7 | 60 |
| 8 | 48 |
| 9 | 36 |
| 10 | 24 |
| 11 | 12 |
| 12 | 0 |

From the partially filled table (assuming no game with different choices) shown below,
it is clear that each firm would produce 48 units.
Then, the price will be 4. (The total quantity demanded is 96)

This Nash equilibrium must be chosen when each firm is producing 48 units, there are both receiving 192 each, and have no incentive to deviate.

| Firm A | | Firm B → | | | | | | | | | | | |
|--------|--------|----------|----------|---------|--------|--------|--------|---------|---------|---------|---------|---------|--------|
| | | 0 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 |
| 0 | 0, 0 | 132, 0 | 120, 120 | 108, 24 | 96, 36 | 84, 48 | 72, 60 | 60, 72 | 48, 84 | 36, 96 | 24, 108 | 12, 120 | 0, 132 |
| 12 | 132, 0 | 120, 120 | 108, 24 | 96, 36 | 84, 48 | 72, 60 | 60, 72 | 48, 84 | 36, 96 | 24, 108 | 12, 120 | 0, 132 | |
| 24 | 144, 0 | 126, 108 | 108, 24 | 96, 36 | 84, 48 | 72, 60 | 60, 72 | 48, 84 | 36, 96 | 24, 108 | 12, 120 | 0, 132 | |
| 36 | 124, 0 | 108, 24 | 96, 36 | 84, 48 | 72, 60 | 60, 72 | 48, 84 | 36, 96 | 24, 108 | 12, 120 | 0, 132 | | |
| 48 | 108, 0 | 96, 36 | 84, 48 | 72, 60 | 60, 72 | 48, 84 | 36, 96 | 24, 108 | 12, 120 | 0, 132 | | | |
| 60 | 84, 0 | 72, 24 | 60, 36 | 48, 48 | 36, 60 | 24, 72 | 12, 84 | 0, 96 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | |
| 72 | 60, 0 | 48, 24 | 36, 36 | 24, 48 | 12, 60 | 0, 72 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | |
| 84 | 48, 0 | 36, 24 | 24, 36 | 12, 48 | 0, 60 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | |
| 96 | 36, 0 | 24, 24 | 12, 36 | 0, 48 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | |
| 120 | 12, 0 | 0, 24 | 0, 36 | 0, 48 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | |
| 132 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | |

* Nash equilibrium

3. (10 points) A monopolist that produces a non-perishable product has the following cost structure per year where all costs are due at the time of the production.

$$TFC = 400$$

$$TVC(Q) = 8Q^2 + 12Q$$

The market demand is currently $P = 300 - 30Q$ per year.

The demand will change every year so as to have 10% higher price at each quantity.

That is, if today (beginning of year 1) monopolist sells 2 units, it will be able to charge $300 - 30 \cdot 2 = \$240$ per unit. Price will be $\$240 \cdot 1.1 = \264 at the end of this year (same as beginning of next year), and, at the end of next year (same as beginning of the following year), it will be $240 \cdot 1.1 \cdot 1.1 = \290.4 etc.

Just like the price, the costs are increasing at 10% per year as well.

Assume that the production and sales can take place only at the beginning or end of any given year, not necessarily at the same time. Each year's production has to be sold by the end of that year. That is, year 1's production cannot be sold in year 2 or 3.

- in different months*
- a) If the monopolist can borrow and lend at 10% per year, when should the production and sale take place in each of the coming three years? (Numerical answer required)

In this case, I would produce at the beginning of the year and sell because I'll be able to make 10% regardless.
if I choose to sell at the beginning of the year, $P = 300 - 30Q$
if I choose to sell at the end, $P = (300 - 30Q) \cdot 1.1$ (next year dollars).
so $P = 300 - 30Q$ in today dollars (same as before) [$P = \frac{300}{1.1}$]

Summary, for the first, $Tc = 400 + 8Q^2 + 2Q$ if I produce today

and $Tc = 400 + 8Q^2 + 12Q$ (per year) if I produce a year later
(same as before) therefore, it doesn't matter when I produce or sell.

- produced and sold at beginning*
- b) If the monopolist can borrow and lend at 12% per year, when should production and sale take place in the coming three years? (can be verbally answered, no numerical answer is required)

I would produce at the end of the year and sell at the beginning
so that I can lend @ 12% of my money as opposed to 10% increase in price
that I would get. Similarly, I would produce at the end to make sure that I can
make 12% by lending even if I have to bear a 10% extra cost in production

- produced at beginning*
- c) If the monopolist can borrow and lend at 8% per year, when should production and sales take place in the coming three years? (can be verbally answered, no numerical answer is required)

I would produce at the beginning and sell at the end so that
(due to 10% increase in price)
I can make a 10% in the market as opposed to losing at 8%. Similarly,
I would produce early to avoid a 10% loss as I am only
making 8% in the market by lending.

P

4. (20 points) LATAE Inc. operating in a monopolistically competitive market has a demand for its product given by $P = 84 - 2Q$ per year. The short run is one year and the corresponding total cost is given by $200 + 5Q^2$. All costs are due at the beginning of the year and the price is charged upfront, that is, obtained at the beginning of the year as well. LATAE's best alternative opportunity provides 10% per year in the market.

An international corporation approaches LATAE, today, to purchase the company at a price of 1,000.

Should LATAE sell the company today? Yes

What would be your answer if LATAE's best investment opportunity were to be 4% per year? No

$$\text{Total revenue at } Q = 0 = (84 - 2 \cdot 0) \cdot 0 = 84 \cdot 0 = 0$$

$$\text{Total cost at } Q = 0 = 200 + 5 \cdot 0^2 = 200$$

$$\text{For } Q > 0, \text{ MR} = MC \Rightarrow 84 - 4Q = 10Q \Rightarrow 14Q = 84 \Rightarrow Q = 6$$

$$\text{Profit} = TR - TC = 84 \cdot 6 - 2 \cdot 36 - (200 + 5 \cdot 36) = 52$$

$$\text{For a market rate of 10\%, opportunity} = \frac{52 + 52}{1.1} = \frac{52 + 520}{1.1} = 572$$

Thus, LATAE SHOULD sell the company ^{today D/F} at a price of 1,000. (Ans 1) [Since 572 < 1000]

If best investment opportunity (market rate) = 4\%.

$$\text{Opportunity} = 52 + \frac{52}{1.04} = 52 + 1300 = 1352$$

Thus, since now the firm's valuation (in dollars) > 1,000, LATAE should NOT sell the company. (Ans 2)

✓ 5. (20 points) You have two investment opportunities:

(a)

I: Invest 10% in the market per year

II: Invest \$50,000 today to get 10,000 every two years beginning a year from today, forever.

Note: Rev. it is payable every 2 years, $r = \frac{1}{(1+0.1)^2} = \frac{1}{1.1^2}$ and $(a = \frac{10000}{1.1^2} \text{ because payment is half yearly, i.e., half rate = 0.05})$

a) Which opportunity would you choose?

If I get \$10,000 every 2 years beginning 1 year from today, forever
Total: $\frac{10000}{1.1} \cdot \frac{10000}{1.1 \cdot 1.1} + \frac{10000}{1.1 \cdot 1.1 \cdot 1.1} + \dots \rightarrow \text{Sum by geometric rule} = \frac{10000}{1.1} \cdot \frac{1}{1 - (\frac{1}{1.1^2})} = \frac{10000}{1.1} \cdot \frac{1}{1 - \frac{1}{1.21}} = \frac{10000}{1.1} \cdot \frac{1.21}{1.21 - 1} = \frac{10000}{1.1} \cdot \frac{1.21}{0.21} = \frac{10000 \cdot 1.21}{1.1 \cdot 0.21} = \frac{12100}{0.21} = 57619.05$ in today dollars.

IR = \$52380.95 \rightarrow NPV = \$52380.95 - \$50000 = \$2380.95 in today dollars.
Since I make economic profit after investing in Scheme 2, then I should choose what I want to choose. [Economic profit = NPV = \$2380.95] Ans \rightarrow choose Scheme 2

b) How much extra would you be willing to invest today in order to get paid \$10,000 per year beginning a year from today, every year?

If I got \$10,000 per year beginning 1 year from today, every year,

Rev: $\frac{10000}{1.1} \cdot 100,000$ (in today dollars)
 $\text{In today dollar} = \frac{10000}{1.1} \cdot 100,000 = 90909.09 \cdot 100,000 = 9090909.09$

To get an economic profit = \$2380.95 \rightarrow I would invest
\$100,000 - \$2380.95 = \$97619.05.

Thus, I would be willing to invest (\$97619.05 - \$50000) = \$47619.05 extra for this deal to get the same economic profit as achieve II. Ans: extra investment = \$47619.05