ENG 110 Winter 2021 Final - Closed Notes/Book

SOLUTIONS

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1. (30 points) In the City of Springfield, Silver Gate Bridge is painted every two years at a cost of $3 million. A startup company came up with a new paint that has the same quality but lasts for 5 years. The City uses a discount rate of 10%.

a) (5 points) Today (year 0), the City is about to pay $3m (not paid yet) when the new paint is offered at a cost of $x million to be paid today and every five years. What should be the maximum x for the City to switch to the new paint?

Solution:

Method 1: Perpetuity

Effective rate for 2 years = $(1.1)^2 - 1 = 0.21$
Effective rate for 5 years = $(1.1)^5 - 1 = 0.61$

Present Value of all the payments (today and in the future) for the original paint = $3m + \$3/0.21 =$ $17.29m

Present Value of all the payments (today and in the future) for the new paint = $xm + \$x/0.61$

New paint will be preferred if $xm + \$x/0.61 < \$17.29$
$x < \$6.55m$

Method 2: Equivalent Uniform Annual Worth Method (distributing costs to end-of-year)

*EUAW or original paint = 3 (A/P,10%,2) = 3*0.5762 = $1.7286*

EUAW of the new paint should be less than $1.7286

$x < 1.7286$ (P/A,10%,5)
$x < 1.7286*3.791$
$x < 6.55$

b) (7 points) Assume that the price of the new paint is set at $x million that you found in part (a) and the City switched to the new paint. Assume that five years passed since the switch. Today (year 5), the City pays $x million but it turns out that the startup company is out of business. Money is paid but the bridge is not painted. Then the City goes back to the original paint and pays $3 million today (year 5) and every two years subsequently. What is the EUAC (Equivalent Uniform Annual Cost) that is paid by the City for having the bridge painted from year 0 to year 9 (excluding the price that is paid exactly at year 9) ?

Solution:

Here is the cash flow of the City (that is, all the payments that the City made from year 0 to 9, excluding the payment at year 9):
$6.55m at year 0
$6.55m at year 5
$3m at year 5
$3m at year 7

NPV of this cash flow is equal to $6.55m+6.55/1.1^5+3/1.1^5+3/1.1^7 = \$14.02m$

EUAW for the first 9 years (distributed to the end of the year) = 14.02 (A/P, 10%, 9) = 14.02*0.1736=\$2.43m

*Alternatively, EUAW for the first 9 years (distributed to the BEGINNING of the year) = (14.02/1.1) (A/P, 10%, 9) = 14.02*0.1736/1.1=\$2.21m*
Graders: both answers are correct (distributed to the end or the beginning of the year!!!)

c) (10 points) The City decides to hire chemical engineers and invest on developing its own paint. Today is year 9 and the total investment required to be paid today is $5 million. This endeavor is expected to produce a paint that will cost $1 million every 3 years, beginning year 13 with 70% chance. With 30% chance the investment will not produce any usable paint (in which case the City will continue to use the original paint). Also, until year 13, the City will use the original paint.

Should the City go ahead with this investment today (year 9)?

Solution:

If the city DOES NOT invest, the present value of the today+future cash flow (calculated at year 9 dollars) = $3m +\$3/.21 = \$17.29m$

If the city does INVEST, the present value of the today+future cash flow (calculated at year 9 dollars)
= $5m + 0.3 ($3m +$3/.21) + 0.7 ($3m+$3m/1.1^2+ ($1m/.33)/1.1) = $15.95m

where the effective rate of 3-years is 1.1^3-1 = 0.33

Since 15.95 < 17.29, the city should INVEST

d)(8 points) What is the Internal Rate of Return of the investment given that this investment will produce a usable paint with certainty (100%)?

Solution:

WE NEED TO TAKE THE CASH FLOW INCREMENT (DIFFERENCE) WITH AND WITHOUT THE INVESTMENT:

Years	Cash Flow without	Cats flow with	Increment
9	-3	-5-3	-5
10			0
11	-3	-3	0
12			0
13	-3	-1	2
14			0
15	-3		3
16		-1	-1
17	-3		3
18			0
19	-3	-1	2
20			0
21	-3		3
22	...	-1	-1

MORE THAN ONE SIGN CHANGE !!! THE IRR CANNOT BE CALCULATED!!!

2. (20 points) A monopolistically competitive company has a Demand given by P = 22,000-6Q, Total Cost given by TC(Q) = 500,000+5Q² .

a) (5 points) What is the profit maximizing price and quantity? What is the profit? Calculate the total consumer surplus amount.

Solution:

$MR = MC$

22,000-12Q = 10Q

$Q^* = 1,000$

Price = 22,000-6*1,000 = 16,000

Profit = 16,000*1,000 - 500,000 - 5* 1,000^2 = 10,500,000

The Consumer Surplus is the Area bordered by the y-axis, demand line, and the price @ 16,000. This is a triangle that has an area of (22,000-16,000)*1,000/2 = 3,000,000

b) (10 points) Due to entry of competitors, firm's demand shifts to a new position that is given by P=22,000-8Q. What is the new profit? How much is the new total consumer surplus ?

Solution:

$MR = MC$

22,000-16Q = 10Q

$Q^* = 846$

Price = 22,000-8*846 = 15,232

Profit = 15,232*846 - 500,000 - 5* 846^2 = 8,807,692

The New Consumer Surplus (22,000-15,232)*846/2 = 2,862,864

c) (5 points) How much would the firm spend on lobby efforts to prevent entry into the industry? How much would the consumer advocacy groups spend to counter these efforts?

Solution:

Firms would be willing to spend the difference in profits: 10,500,000-8,807,692 = 1,692,308

Consumer Groups would spend the difference between the consumer surplus' = 3,000,000 - 2,862,864 = 137,136

3. (20 points) You get a loan of $10,000 today and in each of the following 3 years with an adjustable APR according to which you are charged a 2%, 3%, and 4% interest rate in the first, second and third years respectively. Interest rate will stay at 4% thereafter. You will pay back your debt with 6 equal annual payments beginning 5 years after you get your first loan. How much is your last payment and what portion of it is "the interest payment" and what portion of it is "the principal payment"?

Solution:

The value of debt at year 4 = 10,000*1.02+10,000*1.02*1.03+10,000*1.02*1.03*1.04+10,000 = 41,632

41,632 = A (P/A, 4%, 6) = A * 0.2296
A = $9,559

4. (10 points) A company is funded by $50 million debt with an annual cost is 5% and $50 million by equity with an annual cost of 11%. Tax rate is 10%. What is the after-tax WACC (Weighted Average Cost of Capital) of this company?

Solution:

(1/2)*5%*(1-0.10)+(1/2)*11% = 7.75%

5. (20 points) MARR is 10%. A perfectly competitive firm gets all production orders and is paid the price upfront ($P per unit) for a production process that takes one year. Assume that all costs are due when the production process ends. We know the following about this company:

$TC(Q) = 400,000 + 50Q^3$

If the company makes zero economic profit at the optimum production level (i.e. at the profit maximizing quantity), what is P? (Setting up the equation that will solve for P is enough to get full points. You do not have to solve the equation.)

Solution:

We have two unknowns, P and Q. Therefore, we need two equations.

Metod 1:

Profit = 0
$P*Q$ - 400,000/1.1-50Q^3/1.1 = 0
$MC = P$
150Q^2=P

Method 2:

Profit is zero if the price is at the minimum point of ATC.
Min point of ATC crosses MC and the Price at this point will give a zero profit.
ATC (Q) = P - 400,000/1.1Q - 50Q^2/1.1
ATC(Q) = P - 363,636Q - 45.45Q^2

Derivative of ATC = 400,000/Q^2 - 91Q = 0
Q = (400,000/91)^(1/3) = 16.38
$MC=P$
150Q^2=P