ENG 11
Fall 201

First Midterm

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NAME:_____ ID#:

1. (20 points) Two companies, A and B, are operating in an oligopolistic market and are about to introduce a new product, each with their own format. Assume that each company will simultaneously decide whether to synchronize format and also whether to use freemium revenue model or direct subscription revenue model.

Freemium model requires an investment (COST) of $10 million and subscription model requires $5 million in case of synchronization. Otherwise costs double. Note that, for a synchronized product in the market, both companies need to choose to synchronize. The first table below summarizes the REVENUE for each company depending on their choice of revenue model in case they both choose to synchronize and the second table summarizes the similar information when at least one decides not to synchronize (all numbers are in million $):

SYNC FORMAT:

		Firm B: Subscription	Firm B: Freemium
Firm A	Subscription	68,68 -5,-5	10,70 -5,-10
	Freemium	70,10 -10,-5	35,35 -10,-10

+10, -5

NO SYNC FORMAT:

		Firm B: Subscription	Firm B: Freemium
Firm A	Subscription	42,42 -10,-10	12,50 -10,-20
	Freemium	45,14 -20,-10	20,20 -20,-20

-20, -10 +20

What is the best decision for each company in terms of synchronization and revenue model given that each wants to maximize PROFIT, neither knows exactly what the other will do but each knows the above two tables?

Profits: (Π_A, Π_B)

Firm A \ Firm B	Sync/sub	Sync/free	Nosync/sub	Nosync/free
Sync/subscribe	63,63	5,60	32,32	2,30
Sync/Free	60,5	25,25	25,4	0,0
No sync/sub	32,32	2,30	32,32	2,30
No sync/free	25,4	0,0	25,4	0,0

Turn	Firm	Choice	Result
1	A	Sync/sub	—
2	B	Sync/sub	(63,63)
3	A	Sync/sub	63,63
1	A	Sync/free	—
2	B	Sync/free	(25,25)
3	A	Sync/free	(25,25)
1	A	NS/sub	—
2	B	NS/sub	(32,32)
3	A	NS/sub	(32,32)
1	A	NS/Free	—
2	B	NS/sub	(25,4)
3	A	NS/sub	(32,32)
4	B	NS/sub	(32,32)

Profits (Π)

There are three equilibrium points
(1) Both A & B synchronizing & subscribing: ($63, $63)
(2) Both A & B synchronizing & freemium: ($25, $25)
(3) Both A & B not synchronizing & subscribing: ($32, 32)

Obviously the best choice for both is (1), both sync & subscribing, as that has maximum profits for both firms as the greatest ~~payoff~~ profit made of all equilibrium points.

20

2. (30 points) There are two companies, A and B, operating in a perfectly competitive industry. The respective cost structure for each firm is given below:

A: Total Fixed Cost = 2,900
MC(Q) = 10Q

$TVC_A = \int 10Q\,dQ = 5Q_A^2$ $\quad AVC_A = 5Q_A$

B: Total Fixed Cost = 1,000
MC(Q) = 5Q

$TVC_B = \int 5Q\,dQ = \frac{5}{2}Q_B^2$ $\quad AVC_B = \frac{5}{2}Q_B$

a) There are 1000 firms with the same cost structure as firm A and 500 firms with the same cost structure as B in this industry. If the industry demand is given by P(Q) = 5,000-0.1Q and both A and B already committed to their fixed cost (they already made the purchase and payment for fixed equipment), would either firm choose to produce in the short run?

×8

$P = MR_A = MC_A = 10Q_A \quad Q_A = \frac{P}{10} \quad Q_{A\,industry} = 1000 \cdot \frac{P}{10} = 100P \rightarrow P_{ind_A} = \frac{Q_A}{100}$

$P = MR_B = MC_B = 5Q_B \quad Q_B = \frac{P}{5} \quad Q_{B\,industry} = 500 \cdot \frac{P}{5} = 100P \rightarrow P_{ind_B} = \frac{Q_B}{100}$

$P_{ind\,total} = \frac{Q_A + Q_B}{100}$ let $Q_A + Q_B = Q_{ind}$

$P(Q_{ind}) = \frac{Q_{ind}}{100} = 5000 - 0.1Q_{ind} \rightarrow Q_{ind} = 45{,}454.55$ units

$P(Q_{ind}) = \$454.55$

$Q_A = \frac{P}{10} = 45.45$ units

$Q_B = \frac{P}{5} = 90.91$ units

A: $AVC_A = 5(45.45) = \$227.27$
B: $AVC_B = \frac{5}{2}(90.91) = \$227.27$
with respective $P(Q_{ind})$

Since the industry price > the average variable costs of Firms A & B at their optimal quantities, both should produce in short run

b) Would either firm stay in this business in the long run given there is no new entry into the industry?

×10

$TC_A = 2900 + 5Q_A^2 \quad ATC_A = \frac{2900}{Q_A} + 5Q_A$

$TC_B = 1000 + \frac{5}{2}Q_B^2 \quad ATC_B = \frac{1000}{Q_B} + \frac{5}{2}Q_B$

$ATC_A(45.45) = \$291.07$
$ATC_B(90.91) = \$238.27$ > Both less than $P = \$454.55$

Yes, both firms will stay in long run since $P > ATC$ for both firms.

c) Would either firm stay in this business if in the long run if there is 18,000 more firms entering each having the same cost structure as firm A?

×9

$P_{ind_B} = \frac{Q_B}{100}$ & $Q_{A\,ind} = 19000 \cdot \frac{P}{10} = 1900P \rightarrow P_{ind_A} = \frac{Q_A}{1900}$

$P_{ind_{total}} = \frac{Q_A}{1900} + \frac{Q_B}{100} = \frac{Q_A + 19Q_B}{1900} \rightarrow P_{ind} = \frac{Q_{ind}}{1900} = 5000 - 0.1Q_{ind}$

$Q_{ind} = 49738.22$ units

$Q_{ind} = 95000 + 1900Q$

$P(Q_{ind}) = \$26.19$

$Q_A = \frac{P}{10} = 2.62$ units

$Q_B = \frac{P}{5} = 5.24$ units

Bigger than $P(Q_{ind})$

A: $ATC_A = 5(2.62) + \frac{2900}{2.62} = \$1120.8$

B: $ATC_B = \frac{5}{2}(5.24) + \frac{1000}{5.24} = \$204.09$

Since both ATCs at A & B's optimal Q^* are way higher than the industry price, they shouldn't stay in business long run.

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3. (25 points) Company X is operating in a monopolistically competitive market with a total cost of TC (Q) =1000+4Q². The demand function faced by X is given by P(Q) = 600-100Q.

a) What is the profit maximizing quantity and price for firm X?

$MR = \frac{d}{dQ}[P(Q)\cdot Q] = \frac{d}{dQ}[600Q - 100Q^2] = 600 - 200Q$

$MC = \frac{d}{dQ}TC(Q) = \frac{d}{dQ}[1000 + 4Q^2] = 8Q$

$MR = MC \Rightarrow 600 - 200Q = 8Q$

$600 = 208Q$

$Q^* = 2.88$ units

$P(2.88) = 600 - 100(2.88) = \$311.54$

+5

b) Given that X has already produced the amount found in part a, is it possible to obtain positive profit if the company can identify 10% of its customers who have the highest willingness to pay?

$Q_1 = 0.1 \cdot 2.88 = 0.29$ units $\quad P(0.29) = \$571.15$

$Q_2 = 0.9 \cdot 2.88 = 2.60$ units $\quad P(2.60) = \$340.38$

$\Pi = TR - TC = 571.15 + 340.38 - TC(2.88)$

$= -\$121.75$

+4

Because profit is still negative, we ~~can't~~ lose money & can't obtain positive profit from this price discrimination

c) If price discrimination is not possible, at most how much would firm X pay for an ad campaign that would increase its demand such that the new demand line is given by P(Q) = 1,000-50Q?

$MR = \frac{d}{dQ}[1000Q - 50Q^2] = 1000 - 100Q$

$MC = \frac{d}{dQ}[TC(Q)] = 8Q$

$MR = MC \rightarrow 1000 - 100Q = 8Q$

$1000 = 108Q$

$Q^* = 9.26$

$P(9.26) = \$537.04$

+10

$\Pi_{new} = (537.04)(9.26) - [1000 + 4(9.26)^2] = \$3629.63$

Part (a) → no price discrimination from part b

$\Pi_{old} = (311.54)(2.88) - [1000 + 4(2.88)^2] = -\$134.62$

$\Pi_{difference} = \Pi_{new} - \Pi_{old} = 3629.63 - (-134.62) = \$3764.25$

The firm would be willing to pay at most $3764.25 for ad campaign to avoid a loss (the max value they'll spend before zero economic profit)

Each True/False question is worth 5 points. Explanation with a few sentences is necessary to get full points.

4. TRUE/FALSE – It is not possible to price discriminate unless the firm can identify its customers' willingness to pay either individually or as a group.

Even if the firm can't identify its customers' willingness to pay, it can still employ price discrimination schemes to extract as much money & profit as possible. For example: Say ½ customers will pay high for product A, but not for B, & vice versa for the other half. Say the firm that makes A & B didn't know this. They can still milk money by bundling A & B & selling them at a high but reasonable cost to make more profit.

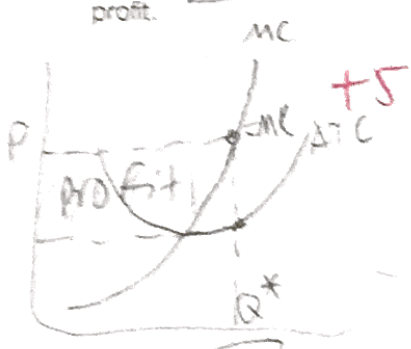
	½	½
A	10	2
B	2	10

×5

5. TRUE/FALSE – If a firm is in the long run and decides to double its size, its total costs will less than double.

×5 This depends on what scale the firm is operating at. If economies of scale this statement is true but if firm is operating at constant or diseconomies ~~regarding return~~ of scale, then not true. Ie: diseconomies means total costs will increase more than double if firm doubles its size

6. TRUE/FALSE – If a company has its marginal cost increasing at an increasing rate, it cannot obtain positive economic profit.



+5

False. ~~Even~~ if the marginal cost is increasing at a fast rate (ie: a quadratic function of quantity Q^2), it does not immediately tell us the total cost of producing at optimal quantity, only tells us the change in cost for adding 1 more unit to produce it. If average total cost is low at Q^* given a high perfect competitive price $P > ATC(Q^*)$, we make profit.

7. TRUE/FALSE – A natural monopoly is highly incentivized to use its positive economic profit for innovation.

+5 There is no incentive for the monopoly to use its positive economic profits to innovate. They are free to choose to if it works out better for them & more profits, but not only is it a risk, it also no incentive to because monopolies own essentially ~~the~~ entire industry and can keep controlling the price & taking all profits from a product, especially since they face no competition or close substitutes to take away their product.

8. TRUE/FALSE – The choice of production scale is a decision that is independent of market demand.

+3 The market demand doesn't dictate how a firm should produce at ~~which scale, as firm owners can choose to change scale anytime. Not only the~~ It's more a decision based on firm's cost structure & revenue, as even in perfect comp, firm A could be able to expand ~~if it's making high economic profit~~ then total costs ~~while~~ increasing scale doesn't increase our total costs by as much, as we experience economies of scale. It's all based on whether scale increases a change in ATC, if economies of scale then ATC decreases, so if 2x scale, TC 23