1:23, 2=25

Midterm 2

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Instructions

- There are **TEN multiple choice questions** and **TWO problems.**
- You should show your work and **box** your answers.
- Books, notebooks, graphical and scientific calculators and phones are not allowed.
- Regulator calculators are allowed.

Discussion 1C

Ksenia

PartI: Multiple Choice Questions (5 points each)

1. The Marshallian demand for good x is given by $x = \frac{p_x}{p_x^2+p_y^2} I$, where p_x denotes the price of good x, p_y the price of good y and I income. The demand elasticity with respect to income is equal to:

   $\frac{\partial x}{\partial I} = \frac{P_x}{P_x^2+P_y^2} \cdot \left(\frac{I}{1} \cdot \frac{P_x^2+P_y^2}{P_x I} \right)$

   1

   (a) 0

   (b) I

   (c) 1

   (d) None of the above

2. A given firm's production function is given by $f(L,K) = (L^\rho + K^\rho)^{\frac{1}{\rho}}$. Its marginal rate of technical substitution is equal to:

   $$\frac{\frac{\partial f}{\partial L} = \frac{1}{\rho}(L^\rho+K^\rho)^{\frac{1}{\rho}-1}(\rho L^{\rho-1})}{\frac{\partial f}{\partial K} = \frac{1}{\rho}(L^\rho+K^\rho)^{\frac{1}{\rho}-1}(\rho K^{\rho-1})}$$

   $\left(\frac{L^{\rho-1}}{K^{\rho-1}}\right)$

   $\frac{1}{2}$ $\frac{1}{6}$

   (a) 0

   (b) $\frac{L}{K}^{\rho-1}$

   (c) $\frac{K}{L}^{\rho-1}$

   (d) None of the above

3. A given firm's production function is given by $f(L,K) = AL^{3\alpha}K^{0.5}$. For what values of α does it exhibit constant returns to scale?

   $3\alpha = 0.5$

   $f(tL, tK) = A(tL)^{3\alpha}(tK)^{0.5}$

   $= At^{3\alpha}L^{3\alpha}t^{0.5}K^{0.5}$

   $= t^{3\alpha+0.5}(AL^{3\alpha}K^{0.5})$

   $3\alpha+0.5=1$

   $3\alpha+\frac{1}{2}=1$

   $3\alpha=\frac{1}{2}$

   (a) $\alpha = 1/3$

   (b) It never exhibits constant returns to scale.

   (c) $\alpha = 0.5$

   (d) None of the above.

4. Assume good x is an inferior good. Which one of the following statements is true:

   S −

   I +

   (a) x is a Giffen good if the income effect is postive.

   (b) The substitution effect and the income effect are of opposite signs.

   (c) The substitution effect and the income effect are of the same sign.

   (d) None of the above

5. Assume a production function $f(L,K) = \min(2L, 6K)$, what combinations of inputs will give the same level of output as $(L,K) = (3,4)$

min(6, 24)

$\min(2L, 6K)$

6

(a) $(L, K) = (3, 7)$ 6, 42
(b) $(L, K) = (7, 1)$ 14 6
(c) $(L, K) = (3, 1)$ 6 6
(d) All of the above.

6. If we observe that the demand for good x decreases when its price increases, we can conclude that:

G : demand ↑
N : I −, S − demand ↓
I : I +, S − demand ?

(a) Good x is a normal good
(b) Good x is an inferior good
(c) Good x is a Giffen good
(d) We can't conclude anything about good x.

7. The compensated price elasticity of demand (hint: the one that uses the Hicksian demand function) accounts for:

$\frac{\partial h_x}{\partial P_y}$

$\frac{\partial g_y}{P_y} = \frac{\partial h_x}{\partial P_y} - x^* \frac{\partial g_x}{\partial I}$

(a) Both income and substitution effects.
(b) Income effect only.
(c) Substitution effect only.
(d) None of the above.

8. The following relationship: $e_{x,p_x} + e_{x,p_y} + e_{x,I} = 1$

$g = \frac{P_x I}{P_x^2 + P_y^2}$

$\frac{(P_x^2 + P_y^2) I - P_x I (2P_x)}{(P_x^2 + P_y^2)^2}$

(a) Is never true
(b) Is always true
(c) Is only true if x is a normal good.
(d) Is only true if both x and y are normal goods.

$\frac{-P_x I(2P_y)}{(P_x^2 + P_y^2)^2} \cdot \frac{P_y(P_x^2+P_y^2)}{P_x I}$

$-\frac{P_x 2P_y}{(P_x^2+P_y^2)}$

$\frac{-2P_y^2}{(P_x^2+P_y^2)}$

9. Assume the price elasticity of demand for cars is equal to -1, while the price elasticity of demand for beer is -0.5. Assume both the price of beer and cars double. Which one of the following statements is true:

$\frac{P_y^2 - P_x^2}{(P_x^2+P_y^2)}$

(a) Both the quantity demanded of beers and cars will increase.
(b) The quantity demanded of cars will decrease more than that of beer
(c) The quantity demanded of cars will decrease less than that of beer
(d) We can't compare how quantities of cars and beer will respond relative to each other because they are in different units.

$-\left(\frac{+P_y^2 - P_x^2}{(P_x^2+P_y^2)}\right)$

(e) You should drink and drive.

10. If capital and labor are perfect substitutes, then:

    (a) the marginal rate of technical substitution is necessarily infinite.
    (b) the marginal rate of technical substitution is necessarily zero.
    (c) the marginal rate of technical substitution is necessarily diminishing.
    (d) the marginal rate of technical substitution is necessarily constant.

$-\frac{I}{(2p_y)}$ $\frac{Ip_x^{-1}}{2}$ $-\frac{Ip_x^{-2}}{2}$ $-\frac{I}{2p_y^2}$

Part II: Essay Questions

Question 1 (25 points) Sami has a preference for good x and good y given by the following utility function:

$$U(x, y) = xy$$

His Marshallian demand functions are given by:

$$g_x = \frac{I}{2p_x} \qquad g_y = \frac{I}{2p_y}$$

His Hicksian demand functions are given by:

$$h_x = \left(\frac{p_y}{p_x}\bar{U}\right)^{0.5} \qquad \left(\frac{p_x}{p_y}\bar{U}\right)^{0.5}$$

The expenditure function is given by:

$$E(p_x, p_y, \bar{U}) = 2(\bar{U}p_x py)^{0.5}$$

The indirect utility function is given by:

$$V(p_x, p_y, I) = \frac{I^2}{4p_x p_y}$$

Where p_x is the price of good x, p_y is the price of good y, I is his income and $\bar{U}$ is a given level of utility.

1. Derive the total effect for good x. (5 points)
2. Derive the substitution effect for good x. Make sure everything is written as a function of I, p_x and p_y ($\bar{U}$ should not appear in the equation). (5 points)
3. Use the Slutsky equation and your answer to question 1 and 2 to derive the income effect for good x. (5 points)
4. Calculate the uncompensated cross price elasticity of good x with respect to good y. (5 points)

5. Assume that when $p_x = 1$ and $p_y = 1$, Sami reaches a utility level of $\bar{U} = 50$. If both prices double, by how much do we need to compensate Sami to keep his utility level at $\bar{U} = 50$ (hint: use the definition of compensating variation). (5 points)

1. Total Effect$_x$ $= \frac{\partial g_x}{\partial P_x} = -\frac{I}{2P_x^2}$

2. Substitution Effect$_x$ $= \frac{\partial h_x}{\partial P_x} = \frac{\partial((P_y \bar{U} P_x^{-1})^{0.5})}{\partial P_x} =$

$0.5(P_y \bar{U} P_x^{-1})^{-0.5}(-P_y \bar{U} P_x^{-2}) = 0.5 P_y^{-0.5} \bar{U}^{-0.5} P_x^{0.5} (-P_y) \bar{U} P_x^{-2}$

$= -0.5 P_y^{0.5} \bar{U}^{0.5} P_x^{-1.5} = -\frac{(P_y \bar{U})^{0.5}}{2P_x^{1.5}}$

$\bar{U} = V(P_x, P_y, I) = \frac{I^2}{4P_x P_y}$

$-\frac{\left(P_y\left(\frac{I^2}{4P_x P_y}\right)\right)^{0.5}}{2P_x^{1.5}} = -\frac{\left(\frac{I^2}{4P_x}\right)^{0.5}}{2P_x^{1.5}} = -\left(\left(\frac{I}{2P_x^{0.5}}\right)\cdot\left(\frac{1}{2P_x^{1.5}}\right)\right)$

$= -\frac{I}{4P_x^2}$

3. $\frac{\partial g_x}{\partial P_x} = \frac{\partial h_x}{\partial P_x} - \frac{\partial g_x}{\partial I} x^* \longrightarrow \frac{\partial g_x}{\partial I} x^* = \frac{\partial h_x}{\partial P_x} - \frac{\partial g_x}{\partial P_x}$

$-\frac{I}{4P_x^2} + \frac{I}{2P_x^2} = \frac{2I}{4P_x^2} - \frac{I}{4P_x^2} = \frac{I}{4P_x^2}$

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Question 1 (answer sheet)

4. $\frac{\partial g_x}{\partial P_y} = 0$

5. $V(1,1,I) = \frac{I^2}{4} = 50 \rightarrow I = 14.14$

$V(2,2,I) = \frac{I^2}{16} = 50 \rightarrow I = 28.28$

$28.28 - 14.14 = 14.14$

Question 2 (25 points)

Assume a firm's production function is given by $f(L, K) = L^{\alpha}K^{1-\alpha}$.

L and K respectively represent the number of units of labor and capital employed by the firm.

The price of labor is given by w and the price of capital by v.

1. Derive the marginal product of labor and the marginal product of capital. (6 points)
2. Derive the marginal rate of technical substitution (RTS). (4 points)
3. Derive the factors of demand L^* and K^* (also known as the cost minimizing input choices) assuming the firm is trying to produce q units of output. (8 points)
4. Assume $\alpha = 0.5$. Derive the total cost function. (3 points)
5. Derive the average cost function. (1 points)
6. Derive the marginal cost function. (1 points)
7. Does this production function exhibit constant, increasing or decreasing returns to scale? Explain and show your work. (2 points)

1. $MP_L = \frac{\partial f}{\partial L} = K^{1-\alpha} \alpha \cancel{L^{\alpha-1}}$ $\quad MP_K = \frac{\partial f}{\partial K} = (1-\alpha)L^{\alpha}K^{-\alpha}$

2. $RTS = \frac{MP_L}{MP_K} = \frac{\alpha L^{\alpha-1} K^{1-\alpha}}{(1-\alpha)L^{\alpha} K^{-\alpha}}$

3. $\mathcal{L} = wL + vK + \lambda(q - L^{\alpha}K^{1-\alpha})$

$\frac{\partial \mathcal{L}}{\partial L} = w - \lambda \alpha K^{1-\alpha}L^{\alpha-1} = 0 \rightarrow \frac{w}{v} = \frac{\cancel{\lambda}\alpha K^{1-\alpha}L^{\alpha-1}}{\cancel{\lambda}(1-\alpha)L^{\alpha}K^{-\alpha}}$

$\frac{\partial \mathcal{L}}{\partial K} = v - \lambda(1-\alpha)L^{\alpha}K^{-\alpha} = 0 \rightarrow$

$\frac{w}{v} = \left(\frac{\alpha}{1-\alpha}\right)\frac{K}{L}$

Question 2 (answer sheet)

$$\frac{w}{v}\left(\frac{1-\alpha}{\alpha}\right)L = K$$

$$q = L^{\alpha}\left(\frac{w}{v}\left(\frac{1-\alpha}{\alpha}\right)L\right)^{1-\alpha}$$

$$q = L\left(\frac{w}{v}\left(\frac{1-\alpha}{\alpha}\right)\right)^{1-\alpha}$$

$$L^{*} = \frac{q}{\left(\frac{w}{v}\left(\frac{1-\alpha}{\alpha}\right)\right)^{1-\alpha}}$$

$$L = \frac{K}{\left(\frac{w}{v}\right)\left(\frac{1-\alpha}{\alpha}\right)}$$

$$q = \left(\frac{K}{\left(\frac{w}{v}\right)\left(\frac{1-\alpha}{\alpha}\right)}\right)^{\alpha} K^{1-\alpha} = \frac{1}{\left(\left(\frac{w}{v}\right)\left(\frac{1-\alpha}{\alpha}\right)\right)^{\alpha}}K$$

$$q\left(\left(\frac{w}{v}\right)\left(\frac{1-\alpha}{\alpha}\right)\right)^{\alpha} = K^{*}$$

Question 2 (answer sheet)

4. $$C = w\left(\frac{q}{\left(\frac{w}{v}\right)^{0.5}}\right) + v\left(q\left(\frac{w}{v}\right)^{0.5}\right)$$

5. $$AC = \frac{w\left(\frac{q}{\left(\frac{w}{v}\right)^{0.5}}\right) + v\left(q\left(\frac{w}{v}\right)^{0.5}\right)}{q}$$

6. $$MC = w\left(\frac{w}{v}\right)^{0.5} + v\left(\frac{w}{v}\right)^{0.5}$$

7. $$f(tL, tk) = t^{\alpha} L^{\alpha} t^{1-\alpha} k^{1-\alpha}$$
$$= t\left(L^{\alpha} k^{1-\alpha}\right)$$
$$= t \cdot f(L, k)$$

Constant, because
$$f(tL, tk) = t(f(L, K))$$