UCLA
Economics 11 – Spring 2018
Professor Lu
MIDTERM 1, Version 1
May 3, 2018

NAME: ___________________________________ UID: __________________

TA Section Number/Time: ____

This is a closed-book exam. There are ten multiple choice questions and two essay questions.
NO calculator is allowed. You have 75 minutes to work on the exam.

Good luck!

Question	Score
Multiple Choice	
Essay Question 1	
Essay Question 2	
Total	

Part I: Multiple Choice Questions (3.5 points each):

1. Which of the following is NOT an axiom required for a utility function to represent preference?

   (a) Continuity

   (b) Completeness

   (c) Homogeneity

   (d) Transitivity

2. If $U(z, r) = 4z + 6r$, then the bundle (4, 2) provides the same utility as:

   (a) (4, 3)

   (b) (3, 2)

   (c) (3.5, 1)

   (d) (2.5, 3)

3. Pizza and coke are complements. Suppose that at current consumption level of 4 pizzas and 3 cokes, an individual's marginal utility of consuming an extra pizza is 12, whereas the marginal utility of consuming an extra coke is 3. Then the *MRS* of coke for pizza (that is, the number of pizzas the individual is willing to give up to get one more coke) is

   (a) 3

   (b) 4

   (c) 1/3

   (d) 1/4

4. What does the transitivity assumption on preferences imply?

   (a) Indifference curves never cross.

   (b) Indifference curves are convex.

   (c) Indifference curves have diminishing *MRS*.

   (d) All of the above.

5. Suppose we have the following equations for the *MRS* of a utility function $U(x, y)$. Which of the following corresponds to a homothetic utility function?

   (a) $MRS(x, y) = xy$

   (b) $MRS(x, y) = \frac{x^2}{y}$

   (c) $MRS(x, y) = 2(x + y)$

   (d) $MRS(x, y) = \frac{x^2+y^2}{xy}$

6. Suppose the utility function is given by $U(\mathrm{x}, \mathrm{y}) = \mathrm{x}^{1/3}\mathrm{y}^{1/5}$, which of the following statement about the utility maximization problem is correct?

   (a) The optimal total spending on good *X* is one-third of total income.

   (b) When the price of *X* increases, and price of *Y* is unchanged, the total consumption of commodity *y* increases.

   (c) The optimal consumption level is the same if the utility function is in the form of $U(\mathrm{x}, \mathrm{y}) = 5ln(\mathrm{x}) + 3ln(\mathrm{y})$.

   (d) When the price of *X* and *Y* both doubles but the income remains the same, then the optimal consumption bundle remains the same.

7. An individual has income *I* and a utility function for skis (x) and ski helmet (y) of the $U(x, y) = \min(2x, y)$. His or her indirect utility function is given by:

   (a) $V = \frac{I}{p_x+p_y}$

   (b) $V = \frac{2p_x+p_y}{I}$

   (c) $V = (2p_x + 2p_y)I$

   (d) $V = \frac{2I}{p_x+2p_y}$

8. There are two types of goods, *X* and *Y*. The price of *X* is 2 and the price of *Y* is 1. At a certain bundle on the budget line, the *MRS* of *X* for *Y* is 3 (i.e., $\frac{-dy}{dx} = 3$), what can you do to increase your utility if your budget constraint remains the same?

(a) Buy less *X* and less *Y*

(b) Buy more *X* and less *Y*

(c) Buy less *X* and more *Y*

(d) Buy more *X* and more *Y*

9. If an individual's utility function is given by $U(x, y) = x^{0.5}y^{0.5}$ and $I = 20$, $p_x = 2$, $p_y = 1$, his or her preferred consumption bundle will be:

(a) $x = 10, y = 20$

(b) $x = 5, y = 10$

(c) $x = 4, y = 8$

(d) $x = 20, y = 5$

10. An individual has a utility function for tennis rackets (x) and tennis balls (y) of the form $U(x, y) = \min(x, y)$. She wants to achieve the utility level $\underline{U}$. Then, her expenditure function is given by:

(a) $E(p_x, p_y, \underline{U}) = (p_x + p_y)\,\underline{U}$

(b) $E(p_x, p_y, \underline{U}) = \frac{\underline{U}}{p_x + p_y}$

(c) $E(p_x, p_y, \underline{U}) = (2p_x + 3p_y)\,\underline{U}$

(d) $E(p_x, p_y, \underline{U}) = \frac{p_x + p_y}{\underline{U}}$

Part II: Essay Questions

Question 1 (30 Points)

John's car has an average of 30 miles per gallon (MPG) when using gas from Allen's station and an average of 40 MPG when using gas from Bob's station. John's preferences are such that he only wants to drive as far as possible given his budget. The gas price is P_a dollars per gallon at Allen's station and P_b dollars at Bob's station. John has a total income of I.

(a) Write down a utility function $U(A, B)$ that represents John's preferences for gallons of gas consumed at Allen's station, A, and gallons of gas consumed at Bob's station B. (Hint: treat A and B as two standard consumption goods and find the utility that describes John's preferences for those goods.) (8 points)

(b) Find John's Marshallian demand functions and the indirect utility function. (Hint: there are three cases.) (8 points)

(c) If $P_a = 3$ and $P_b = 5$, and John's total spending on gas I is 150, what is John's Marshallian demand and how far can John drive? (6 points)

(d) John plans a trip of 1500 miles. He has 10 gallons of gas in his tank from Allen's station, which cannot be resold. John notices that Bob has decreased his price P_b to 3 dollars, whereas the price at Allen's station has not changed. What is the optimal amount of gallons John will buy from Allen's station and Bob's station to complete the trip? How much does he have to spend on gas for the trip? (8 points)

Question 2 (35 Points)

Mary's utility function is given by $U(\mathrm{x}, \mathrm{y}) = \mathrm{x}^{\mathrm{a}}\mathrm{y}^{1-\mathrm{a}}$, the price of *X* and *Y* are given by p_x and p_y, respectively, and total income is equal to *I*.

(a) Write down Mary's budget constraint. (5 points)

For the rest of the question, consider the case in which $\mathrm{a} = 0.2$.

(b) Find Mary's Marshallian demands. You must show your work clearly. (7 points)

(c) What is Mary's total expenditure on good *X*. What is the share of income spent on the good *X*. (7 points)

(d) Suppose now that the price of *X* doubles. Recalculate the share of income spent on the good *X* by Mary. Compare your answer to (d) with that of (c) and explain. (9 points)

(e) Now suppose that Mary's utility function is $U(\mathrm{x}, \mathrm{y}) = \mathrm{x}^{\mathrm{a}+0.3}\mathrm{y}^{1-\mathrm{a}-0.3}$ and that the price of *X* goes back to p_x. Without solving the maximization problem, how many unit of *X* will Mary buy now? (Hint: you can use your findings from part (d).) (7 points)

Extra Blank Space

UCLA
Economics 11 – Spring 2018
Professor Lu
MIDTERM 1, Solutions
May 3, 2018

Part I: Multiple Choice Questions (3.5 points each)

	Version 1	Version 2	Version 3
1	c	b	a
2	d	a	c
3	d	b	c
4	a	b	a
5	d	c	b
6	c	b	d
7	d	c	c
8	b	c	d
9	b	a	b
10	a	c	d

Part II: Essay Questions

Question 1 (30 points)

John's car has an average of 30 miles per gallon (MPG) when using gas from Allen's station and an average of 40 MPG when using gas from Bob's station. John's preferences are such that he only wants to drive as far as possible given his budget. The gas price is P_a dollars per gallon at Allen's station and P_b dollars at Bob's station. John has a total income of *I*.

(a) Write down a utility function U(A, B) that represents John's preferences for gallons of gas consumed at Allen's station, A, and gallons of gas consumed at Bob's station B. (Hint: treat A and B as two standard consumption goods and find the utility that describes John's preferences for those goods.) (8 points)

Because A and B are perfect substitutes, the general form is $U(A, B) = \alpha A + \beta B$.

Since John only cares about the total mileage, so $U(A, B) = 30A + 40B$.

$U(A, B) = 3A + 4B$, $U(A, B) = \frac{3}{4}A + B$, $U(A, B) = \frac{3}{7}A + \frac{4}{7}B$ are also correct.

Note: The answer would have been $U(A, B) = 40A + 30B$ if the problem stated that John is indifferent between 30 gallons of gas from Allen's station and 40 gallons of gas from Bob's station at every combination of *A* and *B*.

(b) Find John's Marshallian demand functions and the indirect utility function. (Hint: there are three cases.) (8 points)

If $\frac{U_A}{P_A} > \frac{U_B}{P_B}$ (equivalently, $\frac{P_A}{U_A} < \frac{P_B}{U_B}$), John should get gas from only Allen's station.

If $\frac{U_A}{P_A} < \frac{U_B}{P_B}$ (equivalently, $\frac{P_A}{U_A} > \frac{P_B}{U_B}$), John should get gas from only Bob's station.

If $\frac{U_A}{P_A} = \frac{U_B}{P_B}$ (equivalently, $\frac{P_A}{U_A} = \frac{P_B}{U_B}$), John is indifferent between Allen's station and Bob's station.

Noting that $MRS = \frac{U_A}{U_B} = \frac{3}{4}$, the following table summarizes the three cases.

		$A^* = g_A(P_A, P_B, I)$	$B^* = g_B(P_A, P_B, I)$	$V(P_A, P_B, I)$
Case (i)	$\frac{P_A}{P_B} < \frac{3}{4}$	$A^* = \frac{I}{P_A}$	$B^* = 0$	$V = \frac{30I}{P_A}$
Case (ii)	$\frac{P_A}{P_B} > \frac{3}{4}$	$A^* = 0$	$B^* = \frac{I}{P_B}$	$V = \frac{40I}{P_B}$
Case (iii)	$\frac{P_A}{P_B} = \frac{3}{4}$	Any $A^* \geq 0$, $B^* \geq 0$ such that $P_A A^* + P_B B^* = I$		$V = \frac{30I}{P_A} = \frac{40I}{P_B}$

(c) If $P_a = 3$ and $P_b = 5$, and John's total spending on gas I is 150, what is John's Marshallian demand and how far can John drive? (6 points)

Since $\frac{P_A}{P_B} = \frac{3}{5} < \frac{3}{4}$, this is case (i).

$A^* = \frac{I}{P_A} = \frac{150}{3} = 50$ gallons. $B^* = 0$ gallon.

30 miles/gallon × 50 gallons = 1500 miles. John can drive 1500 miles.

(d) John plans a trip of 1500 miles. He has 10 gallons of gas in his tank from Allen's station, which cannot be resold. John notices that Bob has decreased his price P_b to 3 dollars, whereas the price at Allen's station has not changed. What is the optimal amount of gallons John will buy from Allen's station and Bob's station to complete the trip? How much does he have to spend on gas for the trip? (8 points)

With the gas in the tank, John can drive 30 miles/gallon × 10 gallons = 300 miles.

So, he needs to buy enough gas to drive 1500 – 300 = 1200 miles.

Since $\frac{P_A}{P_B} = \frac{3}{3} > \frac{3}{4}$, this is case (ii): John buys gas only from Bob's station. Thus, $A^* = 0$.

Since the gas from Bob's station has an average of 40 miles/gallon, John will need to buy $\frac{\text{1200 miles}}{\text{40 miles/gallon}} = 30$ gallons to complete the trip. I.e., $B^* = 30$ gallons.

The gas is priced at \$3/gallon at Bob's station, so the total cost is \$3/gallon × 30 gallons = \$90.

Question 2 (30 points)

Mary's utility function is given by $U(x, y) = x^a y^{1-a}$, the price of *X* and *Y* are given by p_X and p_Y, respectively, and total income is equal to *I*.

(a) Write down Mary's budget constraint. (5 points)

$$I = p_x x + p_y y$$

For the rest of the question, consider the case in which a = 0.2.

(b) Find Mary's Marshallian demands. (7 points)

We can solve this part by using either the Lagrangian method or the shortcut. I will show the solution by using the Lagrangian method.

Since we are finding the Marshallian demands, we should set up the Lagrangian according to Mary's U-Max problem:

$$\mathcal{L} = x^{0.2} y^{0.8} + \lambda(I - p_x x - p_y y)$$

Then, differentiate it with respect to x, y, and λ, and set them all equal to 0.

$$\frac{\partial \mathcal{L}}{\partial x} = 0.2x^{-0.8} y^{0.8} - \lambda p_x = 0 \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial y} = 0.8x^{0.2} y^{-0.2} - \lambda p_y = 0 \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - p_x x - p_y y = 0 \qquad (3)$$

By rearranging and dividing (1) by (2), we will have:

$$\frac{1}{4}\left(\frac{y}{x}\right) = \frac{p_x}{p_y} \Rightarrow y = \frac{4p_x}{p_y} x$$

Then, by plugging this expression into (3), we will get our Marshallian demands:

$$x^* = \frac{I}{5p_x}, y^* = \frac{4I}{5p_y}$$

(c) Calculate Mary's total expenditure on good *X*. Compute the share of income spent on the good *X*. (7 points)

Mary's expenditure on good *X* is: $p_x x^* = \frac{I}{5} = 0.2I$.

The share of income spent on good *X* is computed by dividing her expenditure on good *X* by her income: $\frac{p_x x^*}{I} = \frac{1}{5} = 20\%$

(d) Suppose now that the price of *X* doubles. Recalculate the share of income spent on the good *X* by Mary. Compare your answer to (d) with that of (c) and explain. (9 points)

Let ${p_x}^{new}$ and x^{new} denote the updated price and consumption of good *X*. Therefore, the share of income spent on good *X* by Mary after the price change is:

$$\frac{{p_x}^{new} x^{new}}{I} = \frac{2p_x}{I} \frac{I}{5(2p_x)} = 20\%$$

Notice that the share doesn't change when there is a price change. This is because for Cobb-Douglas functions, the share of income spent on *X* is determined by the exponent a, which is a constant, thus the answer does not change.

(e) Now suppose that Mary's utility function is $U(x, y) = x^{a+0.3}y^{1-a-0.3}$ and that the price of *X* goes back to p_X. Without solving the maximization problem, how many unit of *X* will Mary buy now? (7 points)

As mentioned in the previous part, for a general two goods Cobb-Douglas utility function ($U(x, y) = x^a y^b$), the share of income spent on good *X* is $\frac{a}{a+b}$ and the share of income spent on good *Y* is $\frac{b}{a+b}$. Therefore, we can get Mary's new consumption of good *X* by:

$$\frac{p_x x^*}{I} = \frac{0.5}{1} \Rightarrow x^* = \frac{I}{2p_x}$$