UCLA

Economics 11 – Fall 2009

Professor Mazzocco

MIDTERM 2, Version 1 - Solution

Part 1: Multiple Choice Questions

1) (3 points) Suppose that a firm has a production function of the form $q = f(k,l) = k + 0.5*l^2$ Which of the following statement is true regarding the Marginal Rate of Technical Substitution (RTS)?

a) The RTS depends **both** on k and l.

b) The RTS depends **only on k.**

c) The RTS depends **only on l.**

d) The RTS is **constant**, i.e. it does depend neither on k nor on l.

2) (3 points) Which of the following production functions exhibits CONSTANT returns to scale?

a) $q = f(k,l) = 3*k + 0.5*l$.

b) $q = f(k,l) = 0.5*ln(k) + 0.5*ln(l)$.

c) $q = f(k,l) = k*l$.

d) All of the previous production functions exhibit constant returns to scale.

3) (3 points) If more and more labor is employed while keeping all other inputs constant, the marginal physical productivity of labor will eventually

a) increase.

b) decrease.

c) remain constant.

d) cannot tell from the information provided.

4) (3 points) The marginal physical productivity of capital is defined as

a) a firm's total output divided by total capital input.

b) the extra output produced by employing an additional small unit of capital while allowing other inputs to vary.

c) the extra output produced by employing an additional small unit of capital while holding other inputs constant.

d) the extra output produced by employing an additional small unit of labor while holding capital input constant.

5) (5 points) The Cobb-Douglas production function $q = k^{1/2} l^{3/4}$ yields the cost function $C =$ (where B is a constant, i.e. it does not depend on q, v, w).

a) $Bq\, v^{1/2} w^{3/4}$.

b) $Bq^{5/4} v^{1/2} w^{3/4}$.

c) $Bq^{4/5} v^{1/2} w^{3/4}$.

d) $Bq^{4/5} v^{2/5} w^{3/5}$.

6) (3 points) In this question, use the following definitions for an individual who consumes only two goods:

e_{X,P_X} = price elasticity of demand for X.

e_{Y,P_Y} = price elasticity of demand for Y.

$e_{X,I}$ = income elasticity of demand for X.

$e_{Y,I}$ = income elasticity of demand for Y.

e_{X,P_Y} = cross price elasticity of demand for X.

e_{Y,P_X} = cross price elasticity of demand for Y.

If the consumer's utility function is given by $U(X,Y)=3\ln(X)+2\ln(Y)$, $Px=Py=1$ and $I=10$ than $e_{X,P_X}+e_{X,P_y}+e_{X,I}$ is equal to:

a) 0

b) $\frac{3}{2}$

c) $\frac{2}{3}$

d) -1

7) (5 points) With the Cobb-Douglas utility function $U(x,y)=\sqrt{xy}$, x and y are

a. net and gross substitutes.
b. net substitutes and gross complements.
c. net substitutes and neither gross substitutes nor gross complements.
d. net and gross complements.

8) (3 points) For the cost function $C = 100 + 0.3q$,

a) marginal cost is constant.

b) average cost is U-shaped.

c) average cost increases with q for $q>10$.

d) all of the above are true.

9) (3 points) Technical progress will

a) shift a firm's production function and its related cost curves.

b) not affect the production function, but may shift cost curves.

c) shift a firm's production function and alter its marginal revenue curve.

d) shift a firm's production function and cause more capital (and less labor) to be hired.

10) (3 points) For any given output level, a firm's long-run costs

a. are always greater than or equal to its short-run costs.

b. are usually greater than or equal to its short-run costs except in the case of diminishing returns to scale.
c. are always less than or equal to its short-run costs.
d. are usually less than or equal to its short-run costs except in the case of diminishing returns to scale.

Essay Questions:

1) (46 points) The production function for our new operating system E11 is given by:

$$AK^{\frac{1}{2}}L^{\frac{1}{2}}$$

The price of labor is equal to 2 dollars and the price of capital is equal to 4 dollars.

Consider first the short run decisions of our firm. The current level of capital is fixed at $K = \overline{K}$ and cannot be changed.

a) Write down the cost minimization problem that our firm is solving in the short run. (5 points)
b) Find the contingent demand function for labor and the corresponding short run total cost function. (7 points)
c) Find the short run average cost and the short run marginal cost of labor. (5 points)

Now, let's consider our long run decisions. We can now choose the optimal amount of capital and labor.

d) Write down the cost minimization problem that our firm is solving in the long run. (5 points)
e) Find the contingent demand function for labor and capital and the corresponding total cost function. (7 points)

Suppose that $\overline{K} = 25$, *A=2*, and that we want to produce 100 units of our operating system.

f) How much labor will we hire in the short run? (5 points)
g) How much labor will we hire in the long run? (5 points)

The Obama administration is trying to reduce the unemployment rate by buying 100 units of our product. We will therefore produce 200 units, 100 for the market and 100 for the government.

h) How much labor do we employ in the short and long run in this case? (4 points)
i) Explain why our response to the subsidy in the short run differs from our response in the long run. (3 points)

SOLUTION TO 1)

Let v = 4 and w =2.

a) $\min_L v\bar{K} + wL \ st \quad A\bar{K}^{\frac{1}{2}}L^{\frac{1}{2}} \geq q$.

b) Ls = $(q/A)^2/\bar{K}$ and $Cs = \frac{wq^2}{A^2\bar{K}} + v\bar{K}$.

c) AvCs = $\frac{wq}{A^2\bar{K}} + \frac{v\bar{K}}{q}$ and MgCs = $2\frac{wq}{A^2\bar{K}}$.

d) $\text{Min}_{K,L}\ vK + wL \ st \quad AK^{\frac{1}{2}}L^{\frac{1}{2}} \geq q$.

e) $Kl = \frac{q}{A}\left(\frac{w}{v}\right)^{.5}$, $Ll = \frac{q}{A}\left(\frac{v}{w}\right)^{.5}$, and $Cl = \frac{2}{A}q(vw)^{.5}$.

f) After replacing $\bar{K} = 25$, q=100, and A =2 in b) , we obtain Ls = 100.

g) After replacing q=100, and A =2 in e) , we obtain Ll = 70.71 = $50*2^{.5}$.

h) After replacing $\bar{K} = 25$, q=200, and A =2 in b) and e) , we obtain Ls = 400 and Ll = 141.42 = $100*2^{.5}$.

i) Observe that in the long run the firm has more flexibility to choose the level of capital, whereas in the short run the capital is fixed. This implies that in the short run labor will increase by more (four times as large) than in the long run (twice as large).

2) (20 points) Jean's demand for the good X has the following form:

$$X=300(P_X)^{-2.0}(P_Y)^{1.2}I^{0.8}$$

Where P_X and P_Y are the prices of goods X and Y, respectively, and I is Jean's income.

a) Find the income elasticity of demand for X. Is X a necessity, a luxury, or an inferior good? (5 points)
b) What is the own price elasticity? And what is the cross price elasticity? (5 points)
c) Is X a gross complement or gross substitute with respect to Y? (5 points)

The proportion of income that Jean spends on X is $s=0.5$, that is $P_X X=sI$.

d) Is X a net complement or net substitute with respect to Y? Prove your claim. (5 points)

Solution:

Note:

$$e_{x,I} = \frac{\partial X}{\partial I}\frac{I}{X} = \frac{\partial(lnX)}{\partial(lnI)}$$

$$e_{x,P_x} = \frac{\partial X}{\partial P_x}\frac{P_x}{X} = \frac{\partial(lnP_x)}{\partial(lnI)}$$

$$e_{x,P_y} = \frac{\partial X}{\partial P_y}\frac{P_y}{X} = \frac{\partial(lnP_y)}{\partial(lnI)}$$

and

$$lnX = ln(300) - 2lnP_x + 1.2lnP_y + 0.8lnI$$

Therefore,

a) $e_{x,I} = 0.8 < 1\ (Necessity),$

$this\ implies\ \frac{\partial X}{\partial I} > 0\ (since\ X, I > 0\ ).\ \ X\ is\ a\ Normal\ good$

b) $e_{x,P_x} = -2\ \ and\ \ e_{x,P_y} = 1.2$

c) $Since\ e_{x,P_y} = 1.2, this\ implies\ \frac{\partial X}{\partial P_y} > 0\ \left(since\ X, P_y > 0\ \right).$

$X\ is\ a\ Gross\ Substitute\ of\ Y.$

or

$$\frac{\partial X}{\partial P_y} = 360\frac{P_y^{0.2} I^{0.8}}{P_x^2} > 0\ \left(Since\ P_x, P_y, I > 0\right)\ Gross\ Substitute$$

d) From the Slutsky equation, we know

$$\frac{\partial X}{\partial P_y} = \frac{\partial X^c}{\partial P_y} - Y\frac{\partial X}{\partial I}$$

$$\frac{\partial X^c}{\partial P_y} = \frac{\partial X}{\partial P_y} + Y\frac{\partial X}{\partial I}$$

$$Since\ P_x X = \frac{I}{2}, this\ implies\ P_Y Y = \frac{I}{2}\ \ or\ Y = \frac{I}{2P_Y}$$

$$\frac{\partial X^c}{\partial P_y} = \frac{\partial X}{\partial P_y} + \frac{I}{2P_Y}\frac{\partial X}{\partial I}$$

$$By\ multiplying\ both\ sides\ of\ the\ equation\ by\ \frac{P_Y}{X}$$

$$\frac{\partial X^c}{\partial P_y}\frac{P_Y}{X} = \frac{\partial X}{\partial P_y}\frac{P_Y}{X} + \frac{I}{2P_Y}\frac{\partial X}{\partial I}\frac{P_Y}{X}$$

$$\frac{\partial X^c}{\partial P_y} = \frac{X}{P_Y}(e_{x,P_y} + \frac{1}{2}e_{x,I})$$

$$\frac{\partial X^c}{\partial P_y} = 1.6\frac{X}{P_Y} > 0\ \ Net\ Substitute$$