UCLA

Economic 11 – Fall 2009

Professor Mazzocco

Final Exam, Version 1

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Part I) Multiple Choice Questions

The next two questions refer to an individual whose utility function is given by

$$U(x, y) = x + 3y$$

1. With this utility function, the bundle (4, 1) provides the same utility as the bundle

   a. (1, 4).
   b. (2, 4).
   c. (1, 2).
   d. (2, 1).

2. For this utility function, the *MRS* (*y* for x, i.e. $-\frac{dy}{dx}$)

   a. depends on the values of *x* and *y*.
   b. is always 0.
   c. is always 3.
   d. is always 1/3.

3. If utility is given by $U(X,Y) = \min\{2X, 5Y\}$ and $P_X = 2$, $P_Y = 5$, $I = 60$, this person will choose to consume

   a. (6,15).
   b. (30,0).
   c. (30,12).
   d. (15,6).

4. If utility is given by $U(X,Y) = X + 2Y$ and $\frac{Px}{Py} = 1$, the indirect utility is given by

   a. $\frac{I}{Px}$.

b. $\frac{I}{Px+2Py}$

c. $2\frac{I}{Py}$

d. $2\frac{I}{Px+Py}$.

5. If the utility is given by $U\left(X,Y\right)=X^{\alpha}Y^{1-\alpha}$, the price elasticity and income elasticity of good X will be given, respectively, by

a. 0, $\frac{\alpha}{Px}$
b. -1, 1.
c. $-\frac{\alpha I}{Px^2}, \frac{\alpha}{Px}$.
d. 0, 0.

6. If an individual buys only two goods and these must be used in a fixed relationship with one another (e.g., coffee and cream for a coffee drinker who never varies the amount of cream used in each cup), then

a. there is no substitution effect from a change in the price of coffee.
b. there is no income effect from a change in the price of coffee.
c. Giffen's Paradox must occur if both coffee and cream are inferior goods.
d. an increase in income will not affect cream purchases.

7. Suppose the production function for good q is given by $q=3k+2l$ where k and l are capital and labor inputs. Consider three statements about this function:

I. The function exhibits constant returns to scale.
II. The function exhibits diminishing marginal productivities to all inputs.
III. The function has a constant rate of technical substitution.

Which of these statements is true?

a. All of them
b. None of them
c. I and II but not III
d. I and III but not II

8. For the cost function $C=q^{.8}v^{.4}w^{.6}$ consider the following statements:

I. The function exhibits decreasing average cost.
II. The function is homogeneous of degree 1 in v and w.
III. The elasticity of marginal cost with respect to v exceeds the elasticity with respect to w.

a. None is true.
b. All are true.
c. Only I is true.
d. Only I and II are true.

9. If the demand faced by a firm is perfectly elastic, the marginal revenue of the firm will be equal to

a. 1.
b. 0.
c. Infinity.
d. None of the above.

10. In the short run a perfectly competitive firm's supply curve is given by its marginal cost curve. In order for this to be true, which of the following additional assumptions are necessary

I. That the firm seeks to maximize profits.
II. That the marginal cost curve be positively sloped.
III. That price exceeds average variable cost.
IV. That price exceeds average total cost.

a. All of the above.
b. I and II but not III and IV.
c. I and III but not II and IV.
d. I, II and III, but not IV.

11. If a price-taking firm's production function is given by $q = 1.5\sqrt{k}$, its profit function is given by

a. $9p^2/16v$.
b. $16p^2/9v$.
c. $3p/4v^2$.
d. $3p^2/4v$

12. A firm will hire additional units of any input up to the point where

a. the marginal productivity of the input is maximized.
b. the marginal cost of employing the input is minimized.
c. the expense of employing the last unit is equal to the revenue brought in by the last unit.

d. the revenue brought in by the input is maximized.

13. Which of the following statements is NOT a reason for a shift in the supply curve?

a. Input prices change.
b. Preferences change.
c. Number of producer changes.
d. Technology changes.

14. Let P_X = price of brownies, and P_Y = price of pancakes. Now assume that there are 10 individuals whose demand for brownies is given by $x_1 = 20 - 5P_X + 2P_Y$, and also there are another 20 individuals whose demand $x_2 = 30 - 2P_X + P_Y$. The market demand function for brownies is

a. $X(P_X, P_Y) = 50 - 7 P_X + 3P_y$
b. $X(P_X, P_Y) = 200 - 50 P_X + 20P_y$
c. $X(P_X, P_Y) = 600 - 14 P_X + 20P_y$
d. $X(P_X, P_Y) = 800 - 90 P_X + 40P_y$

15. Firms in a long-run equilibrium in a perfectly competitive industry will produce at the low points of their average total cost curves because

a. free entry implies that long-run profits will be zero no matter how much each firm produces.
b. firms seek maximum profits and to do so they must choose to produce where average costs are minimized.
c. firms maximize profits and free entry implies that maximum profits will be zero.
d. firms in the industry desire to operate efficiently.

The next four questions refer to a market characterized by the following demand and supply functions:

$$Q_D = 10 - 2P$$
$$Q_S = -2 + 4P$$

16. The equilibrium price and quantity in this market are:

a. $(p^* = 2, Q^* = 6)$.
b. $(p^* = 4, Q^* = 6)$.
c. $(p^* = 4, Q^* = 8)$.
d. $(p^* = 3, Q^* = 4)$.

17. (4 points) The consumer surplus is:

a. 12

b. 4
c. 9
d. 15

18. (4 points) The total surplus is:

a. 12
b. 13.5
c. 18
d. 16.5

19. The government decides to impose a price ceiling of $2.5. This price ceiling

a. reduces the total surplus and the consumer surplus
b. reduces the total surplus and the producer surplus
c. does not change the total, consumer, and producer surplus
d. reduces the total, consumer, and producer surplus

20. If the price is lower than the average variable cost, in the short run a profit maximizing firm will

a. produce a positive quantity at which the marginal cost equals the price.
b. exit the market.
c. shutdown production.
d. produce a positive quantity at which marginal revenues are equal to zero.

Part II) Essay Questions

1. (25 points) Ken has a weekly endowment of 100 dollars that he spends on movie (M) and cigars (C). The price of each movie ticket is 10 dollars; the price of cigars is 20 dollars. Joe's utility is given by: U=0.4lnM+0.6lnC.

   a) (5 points) Find his optimal consumption.

      For health reasons, the government decides to impose a tax of $10 on each cigar Ken buys. The tax is paid by Ken.

   b) (7 points) Find the new optimal consumption bundle.
   c) (3 points) How much does the government receive in tax revenues from Ken?

      The government decides to give back to Ken half of the tax revenues collected in the form of a lump sum. Ken has therefore to pay a tax of $10 dollars on each cigar, but receives also a lump sum transfer from the government equal to half the amount computed in part c).

   d) (7 points) Find the new optimal consumption bundle.
   e) (3 points) Is the health policy still effective at reducing the number of cigars Ken smokes? Explain.

Answer:

a) Lagrangian:

$$L = 0.4\ln M + 0.6\ln C + \lambda(100 - 10M - 20C)$$

First order conditions:

$$\frac{\partial L}{\partial M} = \frac{0.4}{M} - \lambda 10 = 0$$

$$\frac{\partial L}{\partial C} = \frac{0.6}{C} - \lambda 20 = 0$$

$$\frac{\partial L(G, M; \lambda)}{\partial \lambda} = 100 - 10M - 20C = 0$$

Rearranging the two first order conditions:

$$\frac{0.4}{10M} = \lambda$$

$$\frac{0.6}{20C} = \lambda$$

Dividing the first expression by the second:

$$\frac{0.4}{0.6} = \frac{10M}{20C} \Rightarrow \frac{M}{C} = \frac{4}{3} \Rightarrow M = \frac{4}{3}C \qquad (1)$$

Substitute (1) into the budget constraint to find:

$$100 = 10\left(\frac{4}{3}C\right) + 20C$$

Solving for the optimal C^* gives:

$$C^* = 3$$

Substituting the optimal C^* back into (1) gives:

$$M^* = 4$$

b) In this case, the price Ken pays is 20+10. The optimality condition becomes:

$$\frac{2C}{3M} = \frac{10}{30} \Rightarrow C = \frac{1}{2}M$$

Plugging back into the budget constraint we get that the optimal consumption of M and C:

$$M = 4, C = 2$$

c) total revenues = 2*10= 20.

d) In this case, the price Ken pays is 20+10 and his income becomes 100+20/2=110. The optimality condition is the same as before:

$$\frac{2C}{3M} = \frac{10}{30} \Rightarrow C = \frac{1}{2}M$$

Plugging back into the budget constraint we get that the optimal consumption of M and C:

$$M = 4.4, C = 2.2$$

e) Yes, but less than before. The income effect reduces the effect of the policy.

2. Suppose there are 120 firms in the market. And each firm has AVC=6q+20, and FC=96.

   a) Calculate the firm's short run supply curve
   b) Calculate the industry short run supply curve.
   c) Suppose that the market demand is given by: Q = -10P + 800. What will the short run equilibrium be (Price, quantity of each firm and the whole industry)
   d) What is the profit of each firm?
   e) Will firms exit or enter the industry?
   f) What is the long run equilibrium in the market (price, quantity and number of firms)?

Answer:

(a) The firm's total cost is TC=AVC*q+FC= $96 + 20q + 6q^2$

In the short run P = MC, P = 12q+20.

Solving for q: q=(P-20)/12

(b) Industry supply (Q)=n*q = 120q

Q = 10(P-20)

(c) Set quantity demanded equal to supply: –10P + 800 = 10(P-20).

P=50, Q=300. For each firm, q=2.5

(d) Profit=Pq-TC=50*2.5-(6*2.5^2+20*2.5+96)=-58.5

(e) Profit is negative, hence some firms will exit the market.

(f) In the long run, AC=MC. 6q+20+96/q=12q+20; q=4

P=AC=68.

Substituting in the demand function:

Q= -10*68+ 800 =120

N=120/4=30.

3. Maurizio owns a company in the mining industry. He hires Sophia, Ken, Gabriela, Paulo, Federico, and Arturo to work for him. Let $X = K^{1/4} \cdot L^{3/4}$.
   a) Derive the cost function.
   b) Find the amount of K and L necessary to produce X=100 when w=2 and v=1.
   c) Sophia asks for a raise for her and her colleagues to w=4. If her request is approved, what would be the new K and L now to produce X=100. Why do you think Sophia and her colleagues would work more or less hours? Is it possible that some of them will be fired?
   d) Find the Marginal and average cost functions.
   e) Find the average product and marginal product for L and K

Answer:

a) Solving the Lagrangian:

$L = wL+rK+ \lambda(X- K^{1/4}L^{3/4})$

FOC:

$$(1)\ \partial L/\partial K = v- \lambda(1/4)K^{-3/4}L^{3/4} = 0$$
$$(2)\ \partial L/\partial L = w- \lambda(3/4)K^{1/4}L^{-1/4} = 0$$
$$(3)\ \partial L/\partial \lambda = X- K^{1/4}L^{3/4}= 0$$

from (1) and (2): $\frac{v}{w} = \frac{1}{3}\frac{L}{K}$, which implies

(4), $K = \frac{wL}{3v}$

Replacing condition (4) in (3) we can get:

$$L = X(3v/w)^{1/4} \quad (5)$$
$$K = X(w/3r)^{3/4} \quad (6)$$

And replacing these in C=wL+rK we can get:

C= $X(3v)^{1/4}w^{3/4} + X(w/3)^{3/4}v^{1/4} = Xw^{3/4}v^{1/4}(3^{1/4} + (1/3)^{3/4})$

b) Replace the values in (5) and (6) $K=100(2/3)^{3/4}$, $L=100(3/2)^{1/4}$

c) $K=100(4/3)^{3/4}$ and $L=100(3/4)^{1/4}$

Labor is more expensive, hence Maurizio will hire less. Sophia and her colleagues will either work part time or some of them will be fired.

d) The marginal cost is defined as ∂C/∂X so in this case is:

MC = AC $= w^{3/4}v^{1/4}(3^{1/4} + (1/3)^{3/4})$

e) MPK = (1/4)$K^{-3/4}L^{3/4}$; APK=$K^{-3/4}L^{3/4}$;
MPL = (3/4)$K^{1/4}L^{-1/4}$ MPK=$K^{-3/4}L^{3/4}$;

4. (50 points) In the southern town of Diesel, there are only two consumers of jeans (J), Ralph and Giorgio. In addition to jeans they also consume the famous local pie (X). Ralph's utility function is $U(J,X) = J(1+X)$, whereas Giorgio's utility function is $U(J,X) = J(2+X)$. Ralph's income is equal to \$10,000 and Giorgio's income is equal to \$21,940. Each pie costs 20\$.

a) (5 points) Find Ralph's and Giorgio's Marshallian demands for jeans.
b) (5 points) Find the market demand for jeans in Diesel.

The market for jeans is perfectly competitive with many identical firms that produce and sell the product in Diesel. Their cost function is $C(J) = 1600 + J^2$.

c) (5 points) Find the number of jeans that each firm produces in a long run equilibrium in this market.
d) (5 points) Find the long-run equilibrium price.
e) (5 points) Compute the equilibrium quantity produced by the entire market.
f) (5 points) How many firms are producing in this market in the long run equilibrium?

Ralph had a great year and his income jumped past Giorgio's at \$28,000

g) (5 points) Find the short run cost function of each firm producing jeans and the short-run market supply for jeans.
h) (5 points) Compute the short run market equilibrium for jeans at the new income levels (price and quantity)
i) (5 points) Is this a long-run equilibrium? Prove your claim
j) (5 points) Find the number of firms that are operating in the market for jeans at the new long-run market equilibrium. (The solution may not be an integer)

Answer:

a) The optimality condition is:

$$\frac{(a+X)}{J} = \frac{p_J}{p_x}$$

where a = 1 for Ralph and 2 for Giorgio.
Solving for X and replacing in the budget constraint, we obtain:

$$J^* = \frac{I + p_x a}{2p_J}$$

Hence,

$$J^* = \frac{10{,}000 + 20}{2p_J}$$ for Ralph and

$$J^* = \frac{21{,}940 + 40}{2p_J}$$ for Giorgio.

b) The market demand is the sum of the two:

$$J^* = \frac{31{,}940 + 60}{2p_J} = \frac{16{,}000}{p_J}$$

c) MC = 2J, AC = $\frac{1600}{J} + J$. Hence, the optimal quantity is the solution of

$$2J = \frac{1600}{J} + J$$

Which is $J^* = 40$.

d) The long run equilibrium price is obtained by substituting the optimal quantity in the AC:

$$p^* = 80$$

e) Substituting in the market demand function, the long run equilibrium quantity is

$$J^* = 16000/80 = 200$$

f) $n^* = 200/40 = 5$

g) $p_J = MC \Rightarrow p_J = 2J \Rightarrow J = \frac{p_J}{2}$

Since there are 5 firms in the market, the market supply is:

$$J_{sr}^S = \frac{5p_J}{2}$$

h) The new market demand is:

$$J^* = \frac{50{,}000}{2p_J} = \frac{25{,}000}{p_J}$$

By setting it equal to the market supply we get the short run equilibrium

$p^* = 100$ and $J^* = 250$

i) No, since each firm has a positive profit ($\pi = 900$)

j) The total quantity supplied and demanded in the long run equilibrium is obtained by substituting the long run equilibrium price in the demand function:

$$J^* = \frac{25{,}000}{p_J} = \frac{25{,}000}{80} = 312.5$$

Hence,

$$n^* = 312.5/40 = 7.8125$$