ECON 101
Professor Vogel
Midterm #1 - Solutions

Question 1 (35 Points): Rob is a member of the UCLA class of 2020. In 2020 he will either work in finance and have a wealth of \$100 in 2020 or work in education and have a wealth of \$36 in 2020. The probability that Rob works in finance is 1/2 while the probability that Rob works in education is 1/2. Rob's utility in 2020 as a function of his wealth in 2020 is $U(w) = w^{1/2}$.

Question 1 part 1 (25 points): What is the certainty equivalent of this lottery for Rob? Call this c.

Question 1 part 2 (10 points): If Sue is risk neutral, would she be willing to sell Rob a contract that promises to pay Rob c in exchange for Rob paying her his wealth (either \$100 if he works in finance or \$36 if he works in education)?

Question 1 part 1 Solution. [10 points for noting this] The certainty equivalent of the lottery, c, is the value c satisfying

$$U(c) = \mathbb{E}[U]$$

where $\mathbb{E}[U]$ is the expected utility of the lottery. [10 points for getting this] The exected utility of the lottery is

$$\mathbb{E}[U] = \frac{1}{2}U(100) + \frac{1}{2}U(36) = \frac{1}{2}100^{1/2} + \frac{1}{2}36^{1/2} = \frac{1}{2}10 + \frac{1}{2}6 = 8$$

Hence, we require

$$U(c) = 8$$

which is equivalent to [5 points for getting this]

$$c^{1/2} = 8 \iff c = 64$$

The certainty equivalent is 64.

Question 1 part 2 Solution. [5 points for noting this] Since Sue is risk neutral, she will be willing to Rob this contract if and only if the expected value of the contract for Sue is positive. [5 points for getting this] The expected value for Sue is

$$\mathbb{E}[V] = \frac{1}{2}(100) + \frac{1}{2}(36) - c = 68 - c = 4$$

Since $\mathbb{E}[V] > 0$ for Sue, she would be willing to sell Rob this contract. [Note to graders: If the students takes a related and correct approach to getting the (correct) answer, give full points.]

Question 2 (65 Points) - A monopoly produces widgets. Demand for widgets is given by

$$p = 9 - q,$$

where p is the firm's price and q is the firm's total quantity. The monopoly currently has one plant in which it can produce widgets. The total cost of producing q_1 widgets in this plant is given by

$$C_1(q_1) = q_1^2$$

The monopoly can choose to build a second plant in which it can produce widgets. The cost of producing q_2 units in the second plant is given by

$$C_2(q_2) = \begin{cases} 0 & \text{if } q_2 = 0 \\ K + q_2^2 & \text{if } q_2 > 0 \end{cases}$$

That is, in order to build this second plant, the monopoly must pay a sunk cost of $K \geq 0$. Solve for the monopoly's total output as a function of K.

Question 2 Solution. We have to figure out if the firm is going to produce in one plant or both. We treat these two cases separately to figure out how much it would produce and how much profit it would make in either case. Then we determine whether it will produce in one or two plants by comparing the profits of the two cases. In both cases, the firm's revenue is $R = q(9-q)$, so that its marginal revenue is $MR = 9 - 2q$.

[25 points for getting the one-plant solution right with 20 for getting $q^{1 \text{ plant}}$ and 5 for getting $\pi^{1 \text{ plant}}$]: If the firm produces in only one plant, it will produce in the existing plant 1. In this case $q = q_1$ and its marginal cost is simply $2q$. The firm will choose q to set $MR = MC$, implying $9 - 2q = 2q$, which implies

$$q^{1 \text{ plant}} = \frac{9}{4}.$$

In this case, the firm's profit is given by

$$\pi^{1 \text{ plant}} = \frac{9}{4}\left(9 - \frac{9}{4}\right) - \left(\frac{9}{4}\right)^2 = \frac{9}{4}\left(9 - \frac{9}{4} - \frac{9}{4}\right) = \frac{9}{4}\left(\frac{9}{2}\right) = \frac{81}{8}.$$

[30 points for getting the two-plant solution right with 5 for getting $q^{2 \text{ plants}}$ and 5 for getting $\pi^{2 \text{ plants}}$]: If the firm produces in both plants, then it must set the marginal cost in plant 1 equal to the marginal cost in plant 2. This requires that it set $q_1 = q_2 = \frac{q}{2}$. Hence, it will split its total output evenly across both plants, implying that its marginal cost of production is only $MC = 2q_1 = 2q_2 = q$. The firm will choose q to set $MR = MC$, implying $9 - 2q = q$, which implies

$$q^{2 \text{ plants}} = 3$$

and this implies

$$q_1^{2 \text{ plants}} = q_2^{2 \text{ plants}} = \frac{3}{2}.$$

In this case, the firm's profit is given by

$$\pi^{2 \text{ plants}} = 3(9-3) - 2\left(\frac{3}{2}\right)^2 - K = 18 - \frac{9}{2} - K = \frac{36-9}{2} - K = \frac{27}{2} - K.$$

[10 points for combining these to get the right answer]: Finally, the firm chooses 2 plants if $\pi^{2 \text{ plants}} \geq \text{or} > \pi^{1 \text{ plant}}$, i.e. if $\frac{27}{2} - K \geq \text{or} > \frac{81}{8}$, i.e. if $K < \text{or} \leq \frac{27}{8}$. Hence, we have

$$q = \begin{cases} \frac{9}{4} & \text{if } K > \text{or} \geq \frac{27}{8} \\ 3 & \text{otherwise} \end{cases}$$