EE3 Fall 2020
Final Exam Practice Problems

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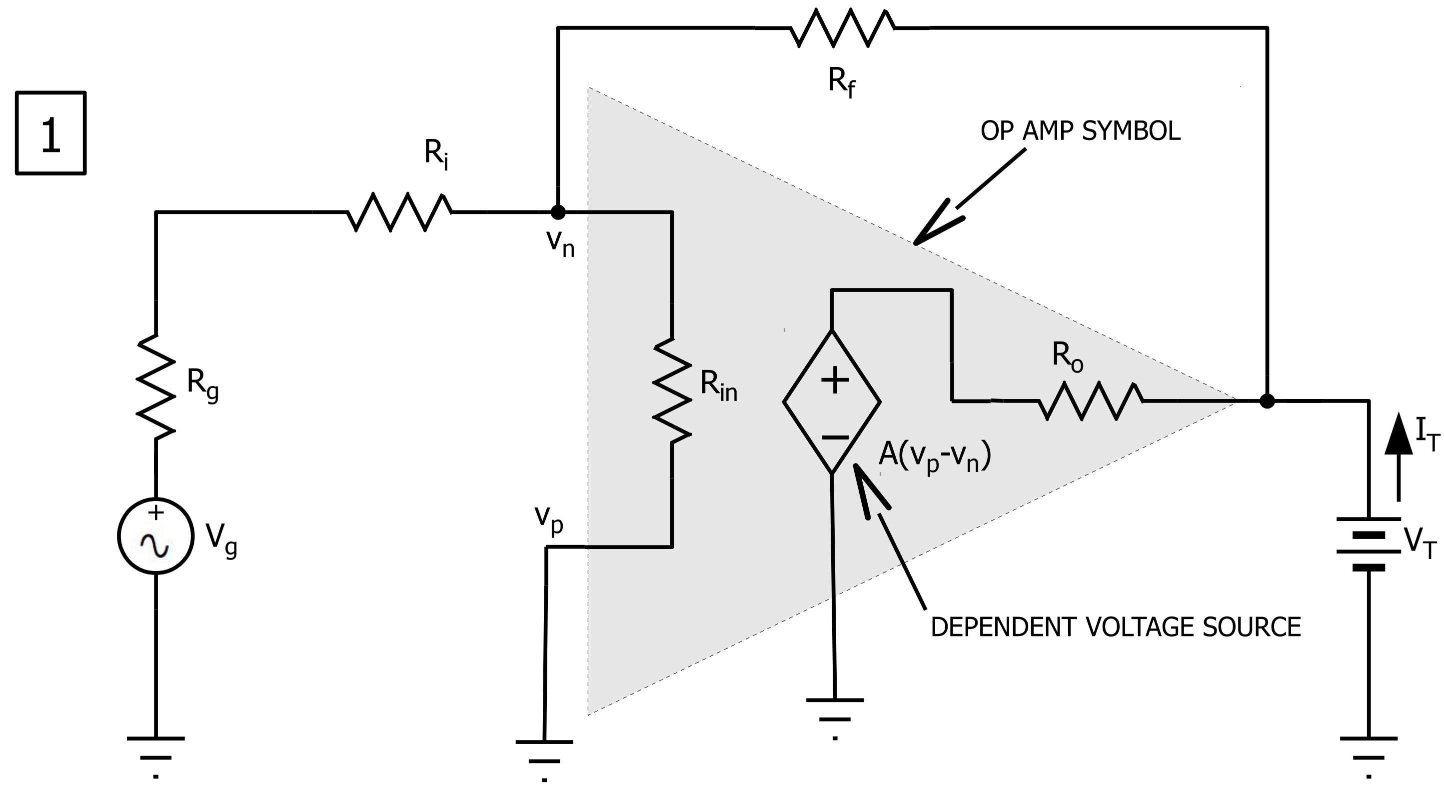
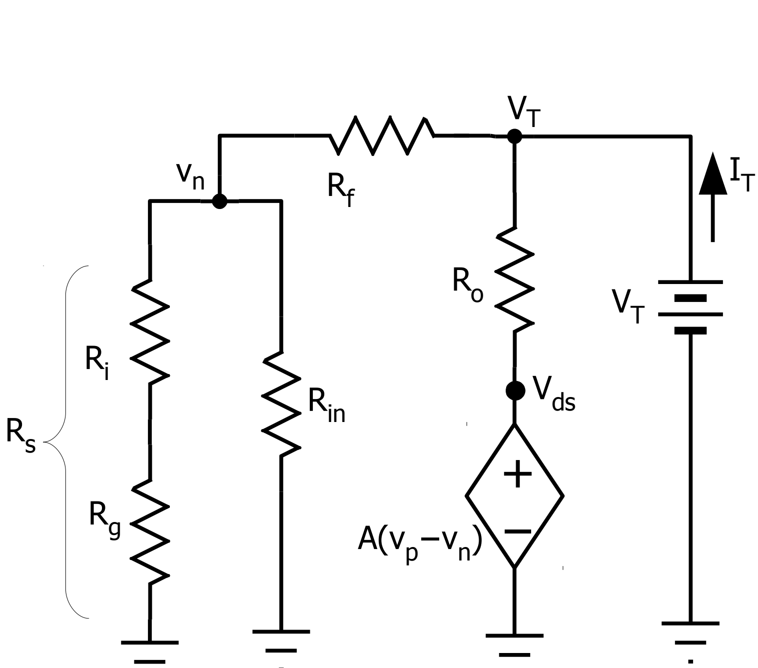


How does one compute the output impedance of an inverting op amp circuit (as opposed to the op amp itself)? One method is to use a slightly-more-realistic equivalent circuit (inside the dashed line of the op amp symbol), and then determine its Thévenin resistance. One can do this by applying a "test" voltage V_T to the output <u>after de-activating the *independent* sources</u>, and then computing the resulting I_T. The ratio of V_T to I_T is the Thévenin resistance and the output impedance‡. Assume that R_g=0, R_i=10 KΩ, R_f=10 KΩ, R_o=10 Ω, R_{in}=1 MΩ, and A = 1e5, and compute the output impedance of the op amp circuit. You will see that the output resistance of the whole circuit is dramatically lower than that of R_o (the op amp itself).



Dependent voltage source: $V_{ds} = A(v_p - v_n) = -Av_n$

① $-I_T + \dfrac{V_T - (-Av_n)}{R_o} + \dfrac{V_T - v_n}{R_f} = 0$

Let $R_s = R_i + R_g$

Then $v_n = V_T\left(\dfrac{R_s \| R_{in}}{R_s \| R_{in} + R_f}\right) = V_T\left(\dfrac{R_s \cdot R_{in}}{R_s \cdot R_{in} + R_f(R_s + R_{in})}\right)$

Let $K = \left(\dfrac{R_s \cdot R_{in}}{R_s \cdot R_{in} + R_f(R_s + R_{in})}\right)$

Then $v_n = V_T K$

Plug into ①:

$$-I_T + \frac{V_T + AV_T K}{R_o} + \frac{V_T(1-K)}{R_f} = 0$$

$$V_T\left(\frac{1}{R_o} + \frac{AK}{R_o} + \frac{1-K}{R_f}\right) = I_T$$

$$R_{out} = R_{th} = \frac{V_T}{I_T} = \frac{1}{\frac{1}{R_o} + \frac{AK}{R_o} + \frac{1-K}{R_f}} = 0.201\text{e-}3\,\Omega << R_o = 10\ \Omega$$

‡ See Nilsson, 8/e, TK454 .N54 2008 (at SEL), Example 4.11

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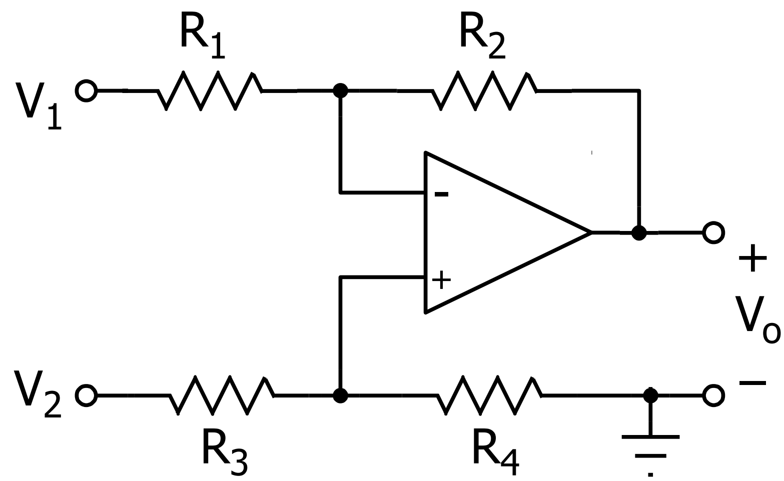
2. Let $R_4 = R_2$, and $R_3 = R_1$. Find $V_o = f(V\text{'s}, R\text{'s})$. HINT: write 2 KCL equations at the input.



$$V_O = \frac{R_2}{R_1}\left(V_2 - V_1\right)$$

Write 2 KCL equations:

$$\frac{v_n - V_1}{R_1} + \frac{v_n - V_o}{R_2} = 0$$

$$\frac{v_p - V_2}{R_3} + \frac{v_p}{R_4} = 0$$

But $R_3 = R_1$ and $R_4 = R_2$, so

$$\frac{v_n - V_1}{R_1} + \frac{v_n - V_o}{R_2} = 0$$

$$\frac{v_p - V_2}{R_1} + \frac{v_p}{R_2} = 0$$

v_n, v_p, and V_o are unknown, so we need a third equation.

$v_p = v_n = v$

$$\frac{v - V_1}{R_1} + \frac{v - V_o}{R_2} = 0 = \frac{v - V_2}{R_1} + \frac{v}{R_2}$$

$$\frac{-V_1}{R_1} - \frac{V_o}{R_2} = -\frac{V_2}{R_2}$$

$$V_o = \frac{R_2}{R_1}\left(V_2 - V_1\right)$$

So this is a *differential amplifier* that provides a controllable gain on the difference between two input voltages.

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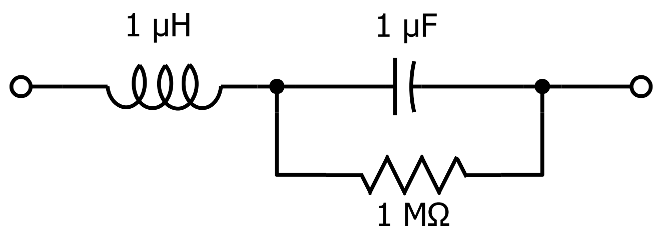


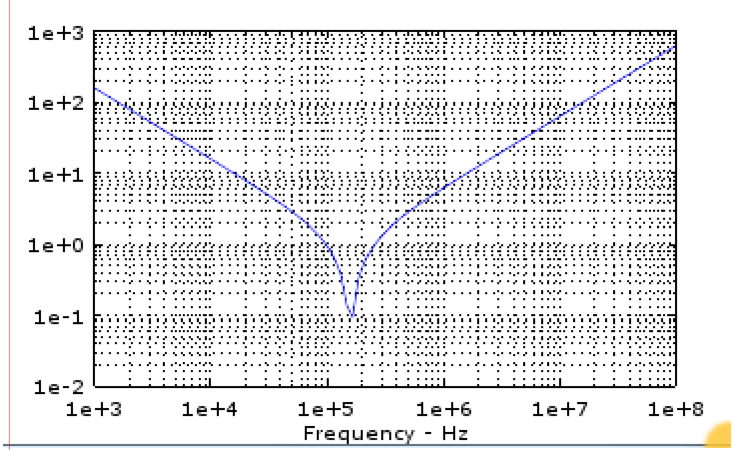
3. This is a more-realistic model of a 1 μF capacitor. The capacitor leads have inductance (1 μH), and the dielectric between the capacitor plates is less than perfect (1 MΩ). Find the frequency where the capacitor stops being a capacitor and starts being an inductor (AKA the series resonant frequency).

At the resonant frequency ω_0, $X_L = X_C$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{(LC)}} = 1\text{e}6 \text{ rad/s)}$$

$$f_0 = \frac{\omega_0}{2\pi} = 159 \text{ KHz}$$



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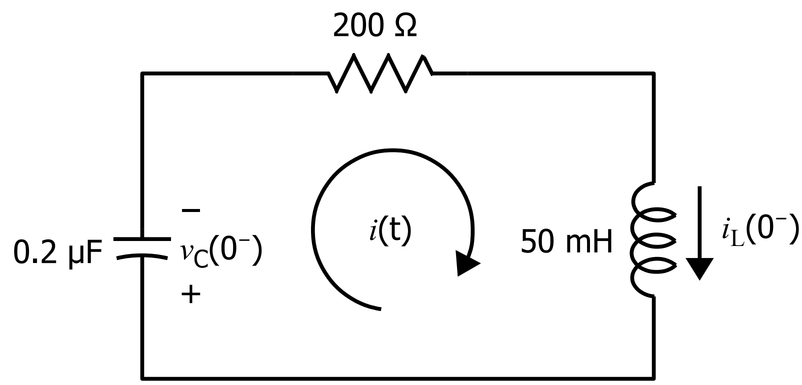
4. This is a second-order circuit. There is an initial voltage on the capacitor (assuming clockwise current) $v(0^-)=+12$ V, and an initial current in the inductor $i_L(0^-)=30$ mA clockwise. In order to solve the differential equation for $i(t)$, the initial voltages $v_L(0^+)$ and $v_R(0^+)$, and $di(t)/dt|_{t=0^+}$ must be found. Using what you know about inductors, capacitors, and KCL/KVL, find these values.



$$i(0^+) = i_L(0^-) = 30 \text{ mA}$$
$$v_C(0^+) = v_C(0^-) = 12 \text{ V}$$
$$v_R(0^+) = i(0^+)R = 0.030 \bullet 200 = 6 \text{ V, + to left}$$
$$\text{KVL@t=0}^+\text{:} +12+6+v_L(0^+) = 0$$
$$v_L(0^+) = -18 \text{ V, + at bottom}$$

Because $v_L(0^+) = L\left[\left.\frac{di(t)}{dt}\right|_{t=0^+}\right]$, it follows that

$$\left.\frac{di(t)}{dt}\right|_{t=0^+} = \left(\frac{1}{0.05}\right)\bullet(-18) = -360 \text{ A/s}$$

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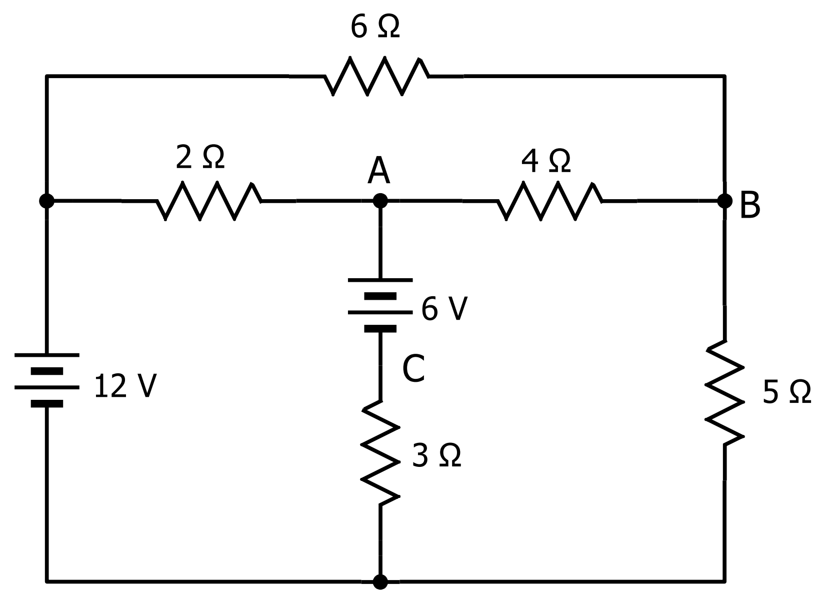
5. Find the power dissipated by the 5 Ω resistor.

9.47 W



$$\frac{V_A - 12}{2} + \frac{V_C}{3} + \frac{V_A - V_B}{4} = 0$$

$$\frac{V_B - V_A}{4} + \frac{V_B}{5} + \frac{V_B - 12}{6} = 0$$

$$V_C = V_A - 6$$

$$0.75\,V_A - 0.25\,V_B + 0.333\,V_C = 6$$

$$-0.25\,V_A + 0.617\,V_B = 2$$

$$V_A - V_C = 6$$

$$V_A = 8.97\ \text{V}$$

$$V_B = 6.88\ \text{V}$$

$$V_C = 2.97\ \text{V}$$

$$P_5 = \frac{(V_B)^2}{5} = \frac{6.88^2}{5} = 9.47\ \text{W}$$

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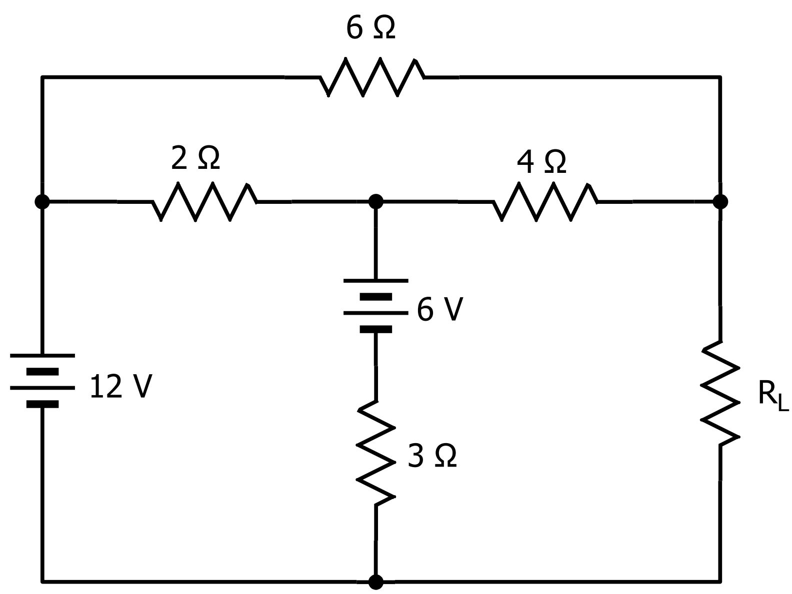
6. Find the value of R_L for maximum power transfer to R_L.



Find the Thévenin Equivalent Resistance.

Open Circuit Voltage

$$V_1 = 12$$

$$\frac{V_2-12}{2}+\frac{V_2-6}{3}+\frac{V_2-V_{OC}}{4} = 0$$

$$\frac{V_{OC}-V_2}{4}+\frac{V_{OC}-12}{6} = 0$$

$$6V_2-72+4V_2-24+3V_2-3V_{OC} = 0$$

$$-3V_2+3V_{OC}+2V_{OC}-24 = 0$$

$$13V_2-3V_{OC} = 96$$

$$-3V_2+5V_{OC} = 24$$

$$V_2 = 9.8571 \text{ V}$$

$$V_{OC} = 10.7143 \text{ V}$$

Short Circuit Current

$$\frac{V_1-12}{2}+\frac{V_1-6}{3}+\frac{V_1}{4} = 0$$

$$V_1 = 7.3846 \text{ V}$$

$$I_4 = \frac{V_1}{4} = 1.8462 \text{ A}$$

$$I_6 = \frac{12}{6} = 2 \text{ A}$$

$$I_{SC} = I_4+I_6 = 3.8462 \text{ A}$$

$$R_{\text{max\_pwr}} = R_{th} = \frac{V_{OC}}{I_{SC}} = 2.79\ \Omega$$

2.79 Ω