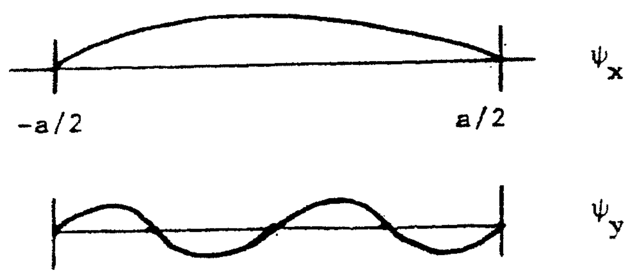
EE2 Professor Harold Fetterman
Feb. 12, 2008 Midterm Closed Book

1a. Light of wavelength λ_1 falls on a metal surface. In this metal surface X_1 **eV** are required to remove an electron. **What is the kinetic energy of a) the fastest and slowest emitted photoelectrons? b) what is the stopping potential? c) What is the cut off wavelength for this metal? Explain, in a few sentences, the main point that this experiment demonstrates.**

1b. We looked at commutators of operators **[A,B]** $\equiv$ **AB-BA**. What are the values of **[p_x , x] and [p_x , y]**. Also find the value of **[E , t]**. **What does it mean for measurements when [A, B] = 0 ?**
What does $\Delta E \Delta t \geq \hbar/2$ mean in terms of a physical system with lifetime Δt ?

2a.In the figure below an electron is in a two dimensional box with the **Ψ_x wavefunction corresponding to an energy of 4 eV.**



What is the total energy of this 2D system? If the Ψ_x wavefunction was raised to an n = 3 level what would the total energy be?

2b. **A energy band is approximately fitted by the expression:**

$$E(k) = E_0(1 - e^{-a^2k^2})$$

Where a is a lattice constant. A) calculate the effective mass a k=o and at the zone boundary k = π/a. Also find the value of k for minimum electron velocity.

3a. Looking at the Density of states for electrons in **two dimensions** we were able to get expressions for the total number of occupied states as a function of energy. **a) Find the value of E_F as a function of n and b) the average energy $\langle E \rangle$ (at T = 0) c) Finally, find the average value of E^2 ($\langle E^2 \rangle$).**

3b. Using our equations for an intrinsic semiconductor and that $n_i = p_i$. **Show that E_{Fi} lies below the middle of the bandgap by kT ln $(m_n^*/m_p^*)^{3/4}$. Also show that the product np is a function of bandgap and is equal to n_i^2**

Now taking n type material ($N_a \approx 0$) the highest operating temperature is when the # of thermally generated carriers n_i is equal to 1/10 the number contributed by impurities $n_i = 0.1\ N_d$. Write an expression for T_{max} .

$$dN_E = f(E)\, Z(E)\, dE$$

$$Z(E) = \frac{4\pi V (2m)^{\frac{3}{2}}}{h^3} E^{\frac{1}{2}}$$

two-dimensional

$$g(\varepsilon)d\varepsilon = \frac{4\pi m A}{h^2}\, d\varepsilon,$$

$$Z(p)\, dp = 8\pi p^2\, dp\ V/h^3$$

$$N = \frac{2Vm^3}{h^3} \int_{v_x}\int_{v_y}\int_{v_z} f_0(v_x, v_y, v_z)\, dv_x dv_y$$

$$dN_p = Z(p)\, dp\, f(E) = \frac{8\pi V}{h^3} \frac{1}{e^{\frac{E-E_F}{kT}} + 1} p^2 dp$$

$$p_x = -i\hbar \frac{\partial}{\partial x} \qquad E = \hbar\omega$$

$$J_x = -ne\langle v_x \rangle \qquad F = ma = \hbar \frac{\partial k}{\partial t}$$

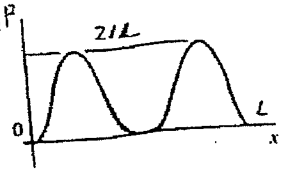
probability of finding a particle is $\Psi^*\Psi\, dx\, dy$

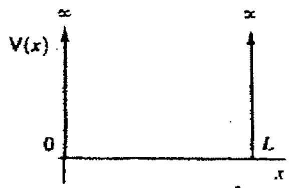
dynamical variable		associated operator
x, y, or z	$\rightarrow$	x, y, or z
$f(x,y,z)$	$\rightarrow$	$f(x,y,z)$
p	$\rightarrow$	$\frac{\hbar}{i}\nabla$
ε	$\rightarrow$	$-\frac{\hbar}{i}\frac{\partial}{\partial t}$

$$v_g = \frac{\partial \omega}{\partial k}$$

$$v = \frac{c}{\lambda}$$

$$a = \frac{\partial v}{\partial k}$$





$$\Delta k = \frac{\text{Force}}{\hbar} \tau_c = \frac{-q\mathcal{E}}{\hbar}$$

$$p_0 = \frac{n_i^2}{n_0}$$

$$E_H = -\frac{m_0 q^4}{2(4\pi\varepsilon_0 \hbar n)^2} = -\frac{13.6}{n^2} \text{eV}, \qquad E_B \simeq -\frac{m_n^* q^4}{2(4\pi K_S \varepsilon_0 \hbar)^2}$$

$$n_0 + N_a = \frac{n_i^2}{n_0} + N_d$$

$$\sigma = q(n\mu_n + p\mu_p)$$

$$h\nu = \phi + eV$$

$$P(x,t)\, dx = \Psi^*(x,t)\Psi(x,t)\, dx$$

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}} \qquad a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{1}{\hbar}\frac{dE}{dk}\right)$$

mobility has the dimension $cm^2V^{-1}sec^{-1}$

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x) = E\psi(x)$$

$$f(E_c) = \frac{1}{e^{\frac{E_c-E_F}{kT}} + 1} \approx e^{-\left(\frac{E_c-E_F}{kT}\right)}$$

$$E = \frac{p^2}{2m}$$

$$\frac{p_x^2}{2m} = \frac{\hbar^2 k_x^2}{2m}$$

$$\frac{1}{\hbar}\frac{d^2E}{dt\, dk} = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt}$$

$$\mathscr{E}(x) = -\frac{dV(x)}{dx}$$

$$dn_E = \frac{4\pi}{h^3} \frac{(2m_c)^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}}\, dE}{\left(e^{\frac{E-E_F}{kT}} + 1\right)}$$

$$p = \sqrt{2m_c(E - E_c)}$$

$$n = \int_{E_c}^{E_{top}} dn_E \qquad J_n = q\mu_n n\mathcal{E} + qD_n \frac{dn}{dx}$$

$$N = \int_0^\infty Z(E) \frac{dE}{e^{(E-E_F)/kT} + 1}$$

$$N_c = 2\left(\frac{2\pi m_c kT}{h^2}\right)^{3/2} \qquad N_v = 2\left(\frac{2\pi m_v kT}{h^2}\right)^{3/2}$$

$$\sigma = \frac{J}{\mathcal{E}}$$

$$\langle E \rangle = \frac{1}{N}\int_0^\infty Z(E) \frac{E\, dE}{e^{(E-E_F)/kT} + 1}$$

$$n = 2\left(\frac{2\pi m_c kT}{h^2}\right)^{\frac{3}{2}} e^{-\left(\frac{E_c-E_F}{kT}\right)} \qquad n_0 + N_a = p_0 + N_d$$

$\frac{kT}{q}$ dimension of volt

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad \frac{\sqrt{2mE_n}}{\hbar} = \frac{n\pi}{L}$$

$$n = N_c e^{-\left(\frac{E_c-E_F}{kT}\right)} \qquad N_c = 2\left(\frac{2\pi m_c kT}{h^2}\right)^{\frac{3}{2}}$$

$$k = n\pi/L, \qquad n = 1, 2, \ldots \qquad E = E_x + E_y + E_z$$

$$E_n = n^2\hbar^2\pi^2/2mL^2 = n^2h^2/8mL^2$$

$$p = 2\left(\frac{2\pi m_v kT}{h^2}\right)^{\frac{3}{2}} e^{-\left(\frac{E_F-E_v}{kT}\right)} \qquad p = n + N_A^-$$

$$n_0 p_0 = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

$$E_{n_x n_y n_z} = (\hbar^2\pi^2/2mL^2)(n_x^2 + n_y^2 + n_z^2)$$

$$N_D^+ = N_D\left(1 - \frac{1}{e^{\left(\frac{E_D-E_F}{kT}\right)} + 1}\right) \qquad N_A^- = \frac{N_A}{1 + e^{\frac{E_A-E_F}{kT}}}$$

$$n_i = N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right]$$

$$p_i = N_v e^{-(E_i - E_v)/kT}$$

$$n_i^2 = n_i \times p_i \qquad \psi_x(x) = \psi_x(x + L_x)$$

$$\psi_x = A \sin kx, \quad k = \frac{\sqrt{2mE}}{\hbar} \qquad k = \frac{n\pi}{L},$$

$$n_i = \sqrt{N_c N_v}\, e^{\frac{-E_g}{2kT}} \qquad \mathcal{E} = -\frac{d\psi}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

$$\psi(x,y,z) = \psi_x(x)\, \psi_y(y)\, \psi_z(z)$$

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Midterm Exam 2/12/08

1a) $h\nu = \phi + eV \qquad \nu = \frac{c}{\lambda}, \; \phi = \chi_1$

$KE = \frac{hc}{\lambda_1} - \chi_1$ max electron.

$KE - min = 0$ electron deep in metal.

b) $V = \frac{\frac{hc}{\lambda_1} - \chi_1}{e}$ stopping voltage.

c) $\frac{hc}{\lambda} - \chi_1 = 0 \qquad \lambda_{cut\,off} = \frac{hc}{\chi_1}$

This photoelectric experiment shows that light is composed of photons - and that the energy of these 'photons' is equal to $h\nu$.

1b) $[A,B] = AB - BA.$

$p_x = -i\hbar\frac{\partial}{\partial x} \quad [p_x, x] =$

$-i\hbar\frac{\partial}{\partial x}(x) - x\left(-i\hbar\frac{\partial}{\partial x}\right)$

product.

$-i\hbar - i\hbar x\frac{\partial}{\partial x} + xi\hbar\frac{\partial}{\partial x} = -i\hbar$

now for $[p_x, y] \rightarrow -i\hbar\frac{\partial}{\partial x}y + i\hbar y\frac{\partial}{\partial x}$

$= 0$ if $[A,B]=0$ then it is possible to measure both A, and B quantities simultaneously. Otherwise the uncertainty principle holds.

$[E,t] = -\frac{\hbar}{i}\frac{\partial}{\partial t}(t) - t\left(-\frac{\hbar}{i}\frac{\partial}{\partial t}\right) = -\frac{\hbar}{i} \cancel{- \frac{\hbar}{i}t\frac{\partial}{\partial t}} \cancel{+ \frac{\hbar}{i}t\frac{\partial}{\partial t}}$

so $[E, t] = i\hbar$

$\Rightarrow \Delta E \Delta t \geq \frac{\hbar}{2}$ uncertainty Principle holds.

$\therefore$ lifetime Δt means that the energy is uncertain

$$\Delta E \geq \frac{\hbar}{2\Delta t}.$$

2a) $E = \frac{\hbar^2 \pi^2 n^2}{2mL^2} = \frac{\hbar^2 \pi^2 n^2}{2ma^2}$

$E_T = E_x + E_y \quad E_x = 4$ and $E_y = 4n^2$

$n = 4.$

~~so~~ $E_T = 4 + 16(4) = 68\text{eV}$

$\longrightarrow$ If $E_y = (3)^2(4) = 36$ then $E_T = 36 + 64 = 100\text{eV}.$

2b) $E(k) = E_0(1 - e^{-a^2k^2}) \quad \frac{\partial E}{\partial k} = E_0(-2a^2k)(-e^{-a^2k^2})$

$\frac{\partial^2 E}{\partial k^2} = E_0(-2a^2)(-e^{-a^2k^2}) + E_0(-2a^2k)^2(e^{-a^2k^2})$

$= E_0(+e^{-a^2k^2})(2a^2 - 4a^4k^2)$

a) $k=0 \quad m^* = \frac{\hbar^2}{2a^2E_0}$

b) $k = \frac{\pi}{a}$

$$m^* = \frac{\hbar^2}{e^{-\pi a}(2a^2 - 4a^2\pi^2)}$$

2b cont.

$$\frac{\partial^2 E}{\partial k^2} = E_0\left(e^{-a^2k^2}\right)\left(2a^2 - 4a^4k^2\right) = 0$$

for max velocity as a function of k.

$$2a^2 - 4a^4k^2 = 0 \qquad k = \pm\frac{1}{\sqrt{2a^2}}$$

$$1 - 2a^2k^2 = 0$$

$$k = \pm\frac{1}{a\sqrt{2}}$$

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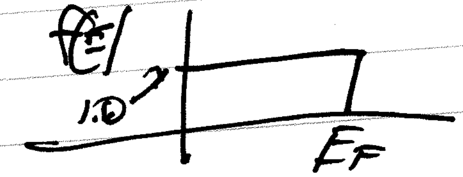
3a) $Z(E)\,dE = g(E)\,dE. = \frac{4\pi m A}{h^2}\,dE$

$$n = \int_0^{E_F} \frac{4\pi m A}{h^2}\,dE$$

since f(E) is a step function.



then for 2D

$$\frac{E_F\,4\pi m A}{h^2} = N \qquad E_F = \frac{h^2}{4\pi m}\frac{N}{A}.$$

$$n = \frac{N}{A} \qquad E_F = \frac{n h^2}{4\pi m A}.$$

b) $\langle E\rangle = \dfrac{\int_0^{E_F} Z(E)\,E\,dE}{N} = \dfrac{C\int_0^{E_F} E\,d}{C\int_0^{E_F} d}$

$$\langle E\rangle = \frac{E^2}{2}\Big|_0^{E_F} \Rightarrow \frac{E_F}{2}$$

c)

$$\langle E^2\rangle = \frac{C\int_0^{E_F} E^2\,dE}{C\int_0^{E_F} dE} \Rightarrow \frac{\frac{E_F^3}{3}}{E_F}$$

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$$\langle E^2\rangle = \frac{E_F^2}{3}.$$

3b) $n_i = N_c e^{-(E_c - E_{Fi})/kT} = N_v e^{-(E_{Fi} - E_v)/kT}$

$= p_i$

so $\frac{N_c}{N_v} = e^{\frac{-(E_{Fi} - E_v)}{kT}} e^{\frac{E_c - E_{Fi}}{kT}}$

$$\ln \frac{N_c}{N_v} = \frac{-(E_{Fi} - E_v)}{kT} + \frac{(E_c - E_{Fi})}{kT}$$

$$= -\frac{2E_{Fi}}{kT} + \frac{E_c + E_v}{kT} = \ln\frac{N_c}{N_v}.$$

$$\Rightarrow E_{Fi} = \frac{E_c + E_v}{2} - kT \ln \frac{2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}}{(2)\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}}$$

$\uparrow$ middle gap.

$$E_{Fi} = \frac{E_c + E_v}{2} - \frac{kT}{2} \ln \left(\frac{m_n^*}{m_p^*}\right)^{3/2}$$

$$= \frac{E_c + E_v}{2} - kT \ln \left(\frac{m_n^*}{m_p^*}\right)^{3/4}$$

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$$np = N_v N_c e^{\frac{-(E_c - E_v)}{kT}} = \sqrt{N_c N_v}\, e^{\frac{-E_g}{kT}}$$

$$= n_i^2$$

$$n_i = \sqrt{N_c N_v}\, e^{\frac{-E_g}{2kT}} = 0.1\, N_D$$

$$\ln\left(\frac{0.1\, N_D}{\sqrt{N_c N_v}}\right) = \frac{-E_g}{2kT}$$

$$T_{max} = \frac{-E_g}{2k \ln\left(\frac{.1 N_d}{\sqrt{N_c N_v}}\right)}$$

$$= \frac{E_g}{2k \ln\left(\frac{\sqrt{N_c N_v}}{.1 N_d}\right)}$$