EE2 Midterm Examination
Prof. H. R. Fetterman 02/8/07
Please do all work on separate sheets

1a. For the indicated planes find the Miller indices. Please show your work.

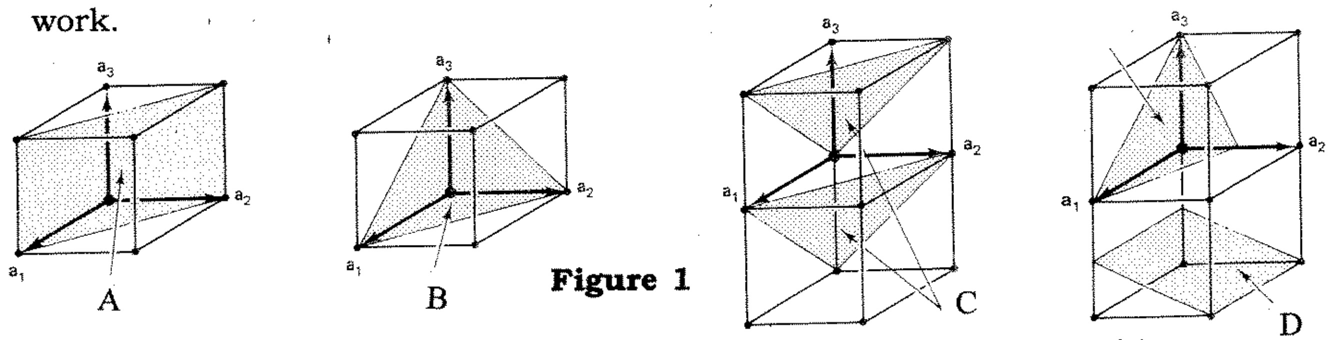


Figure 1

1b. In the Figure 2 below an electron is in a one dimensional box with the **Ψ_I wavefunction** corresponding to an energy of **4 eV. What is the energy in the state corresponding to Ψ_{II} ?** What is the lowest possible energy for an electron is this box?

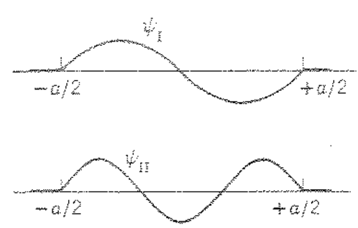


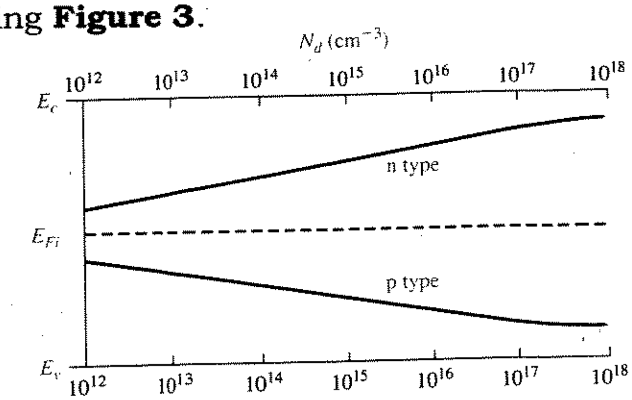
Figure 2

1c. An energy band is approximated by the expression:

$$\mathbf{E(k) = (\hbar^2 k^2/2m_o) - Ak^4}$$

Using the condition that $\mathbf{v_g = 0}$ **at** $\mathbf{k = \pi/a}$ calculate **A.** Also calculate $\mathbf{m^*}$ **when k = 0** and **k = π/a**. What is the value of k **for which $\mathbf{v_g}$ is maximum?**

2a. Using the equations for p and p_i write an expression for the difference between the Fermi level **(E_F)** and the intrinsic Fermi energy **(E_i)** in p type material. Explain the following **Figure 3**.



2b. Write the expression for charge neutrality keeping both terms $\mathbf{N_D}$ and $\mathbf{N_A}$. Now solve the resulting quadratic equation to find an expression for **n**. Show that in a compensated n type (large N_D and N_A) the minority hole concentration : $\mathbf{p_n = n_i^2 / n_n = n_i^2/(N_D - N_A)}$

2c. Explain why **As** is a donor in **Si material** and why **Ga** is a acceptor in **Si material.** Now explain what would happen if **Si** atoms were introduced into **GaAs** material.

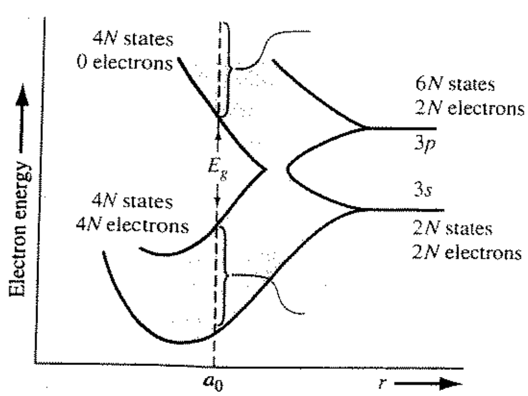
In terms of the donor atoms estimate their **ionization energy** if the dielectric constant of the material is A and the effective mass of an electron is $\mathbf{0.Bm_e}$.

$Z(p)dp = 4L\,dp/h$

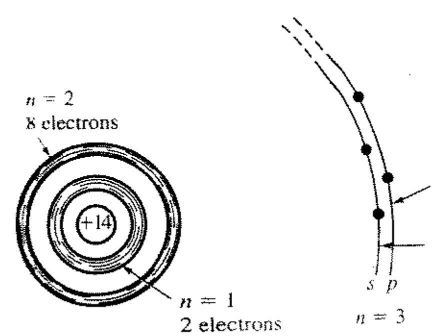
3a. Using the density of state function $\mathbf{Z(p) = 4Ldp/h}$ in 1 dimension find the average **p at T = 0** in terms of $\mathbf{p_F}$. Now derive an expression for the average value of $\mathbf{p^2}$. What is the **average kinetic energy** based upon this value?

3b. Convert the density of electrons, per unit volume, **Z(p)dp** to a function of energy **Z(E)dE in 1 dimension**. Now derive the total number of electrons **(n),** per unit length, in a metal as a **function of the Fermi energy at T = 0.** Write an expression for the **Fermi energy as a function of n.** How would the Fermi Energy change if the effective mass was **reduced by a factor of 2?**

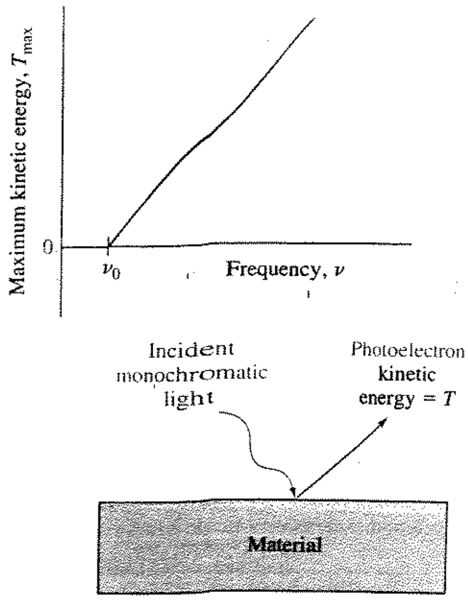
3c. Finally, please explain the following diagrams in **Figure 4**, with equations, in no more than **three sentences each**.



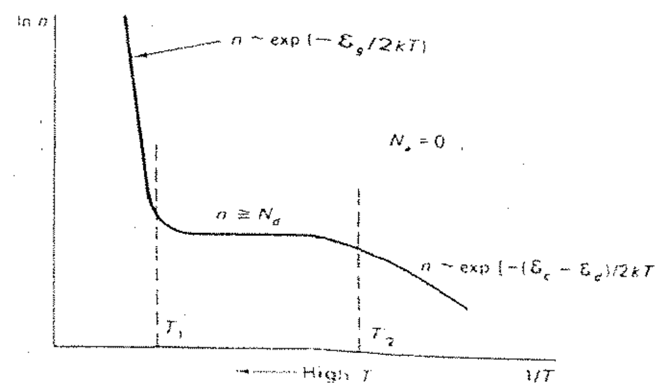
A



B


Maximum kinetic energy, T_{max}
0
ν_0
Frequency, ν
Incident monochromatic light
Photoelectron kinetic energy = T
Material

C


ln n
$n \sim \exp(-\mathcal{E}_g/2kT)$
$N_a = 0$
$n \cong N_d$
$n \sim \exp[-(\mathcal{E}_c - \mathcal{E}_d)/2kT]$
T_1
T_2
High T
$1/T$

D

$$\phi_{av} = \int \frac{\phi\, dN_E}{N}$$

$$Z(E) = \frac{4\pi V(2m)^{\frac{3}{2}}}{h^3} E^{\frac{1}{2}}$$

$$Z(p)\,dp = 8\pi p^2\,dp\; V/h^3$$

$$dN_E = f(E)\,Z(E)\,dE$$

$$N = \frac{2Vm^3}{h^3}\int_{v_x}\int_{v_y}\int_{v_z} f_0(v_x,v_y,v_z)\,dv_x\,dv_y$$

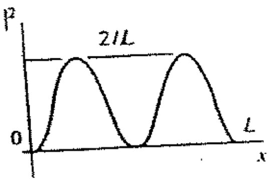
$$dN_p = Z(p)\,dp\,f(E) = \frac{8\pi V}{h^3}\frac{1}{e^{\frac{E-E_F}{kT}}+1}p^2dp$$

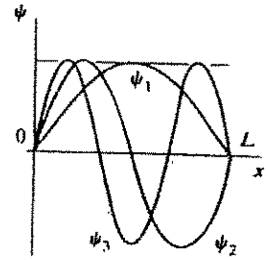
$$p_x = -i\hbar\frac{\partial}{\partial x} \qquad E = \hbar\omega$$

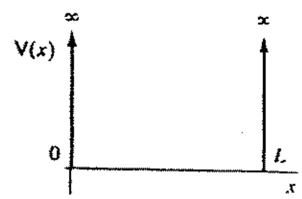
probability of finding a particle is $\Psi^*\Psi\,dx\,dy$

$$J_x = -ne\langle v_x \rangle \qquad F = ma = \hbar\frac{\partial k}{\partial t}$$

$$\nu = \frac{c}{\lambda} \qquad a = \frac{\partial v}{\partial k}$$







$$v_g = \frac{\partial \omega}{\partial k}$$

$$\Delta k = \frac{\text{Force}}{\hbar}\tau_c = \frac{-q\,\mathcal{E}}{\hbar}$$

$$p_0 = \frac{n_i^2}{n_0} \qquad n_0 + N_a = \frac{n_i^2}{n_0} + N_d$$

$$E_H = -\frac{m_0 q^4}{2(4\pi\varepsilon_0\hbar n)^2} = -\frac{13.6}{n^2}\text{ eV}, \qquad E_B = -\frac{m_n^* q^4}{2(4\pi K_S\varepsilon_0\hbar)^2}$$

$$\sigma = q(n\mu_n + p\mu_p)$$

$$h\nu = \phi + eV \qquad P(x,t)\,dx = \Psi^*(x,t)\Psi(x,t)\,dx$$

mobility has the dimension $\text{cm}^2V^{-1}\text{sec}^{-1}$

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}} \qquad a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{1}{\hbar}\frac{dE}{dk}\right)$$

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x) = E\psi(x)$$

$$\frac{p_x^2}{2m} = \frac{\hbar^2 k_x^2}{2m} \qquad f(E_c) = \frac{1}{e^{\frac{E_c-E_F}{kT}}+1} \approx e^{-\left(\frac{E_c-E_F}{kT}\right)} \qquad E = \frac{p^2}{2m}$$

$$\frac{1}{\hbar}\frac{d^2E}{dt\,dk} = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt}$$

$$\mathscr{E}(x) = -\frac{dV(x)}{dx} \qquad dn_E = \frac{4\pi}{h^3}\frac{(2m_c)^{\frac{3}{2}}(E-E_c)^{\frac{1}{2}}\,dE}{\left(e^{\frac{E-E_F}{kT}}+1\right)}$$

$$p = \sqrt{2m_c(E-E_c)}$$

$$n = \int_{E_c}^{E_{top}} dn_E \qquad J_n = q\,\mu_n\,n\,\mathcal{E} + q\,D_n\frac{dn}{dx}$$

$$N_c = 2\left(\frac{2\pi m_c kT}{h^2}\right)^{3/2} \qquad N_v = 2\left(\frac{2\pi m_v kT}{h^2}\right)^{3/2} \qquad \sigma = \frac{J}{\mathcal{E}}$$

$$N = \int_0^\infty Z(E)\frac{dE}{e^{(E-E_F)/kT}+1}$$

$$\langle E\rangle = \frac{1}{N}\int_0^\infty Z(E)\frac{E\,dE}{e^{(E-E_F)/kT}+1}$$

$$n = 2\left(\frac{2\pi\,m_c\,kT}{h^2}\right)^{\frac{3}{2}} e^{-\left(\frac{E_c-E_F}{kT}\right)} \qquad n_0 + N_a = p_0 + N_d$$

$\frac{kT}{q}$ dimension of volt

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad \frac{\sqrt{2mE_n}}{\hbar} = \frac{n\pi}{L}$$

$$n = N_c\,e^{-\left(\frac{E_c-EF}{kT}\right)} \qquad N_c = 2\left(\frac{2\pi\,m_c\,kT}{h^2}\right)^{\frac{3}{2}}$$

$$k = n\pi/L, \qquad n = 1, 2, \ldots \qquad E = E_x + E_y + E_z$$

$$p = 2\left(\frac{2\pi\,m_v\,kT}{h^2}\right)^{\frac{3}{2}} e^{-\left(\frac{E_F-E_v}{kT}\right)} \qquad p = n + N_A^-$$

$$E_n = n^2\hbar^2\pi^2/2mL^2 = n^2h^2/8mL^2$$

$$n_0 p_0 = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

$$E_{n_x n_y n_z} = (\hbar^2\pi^2/2mL^2)(n_x^2 + n_y^2 + n_z^2)$$

$$N_D^+ = N_D\left(1 - \frac{1}{e^{\left(\frac{E_D-E_F}{kT}\right)}+1}\right) \qquad N_A^- = \frac{N_A}{1 + e^{\frac{E_A-E_F}{kT}}} \qquad n_i = N_c\exp\left[\frac{-(E_c - E_{Fi})}{kT}\right]$$

$$p_i = N_v e^{-(E_i-E_v)/kT}$$

$$n_i^2 = n_i \times p_i \qquad \psi_x(x) = \psi_x(x + L_x)$$

$$\psi_x = A\sin kx, \qquad k = \frac{\sqrt{2mE}}{\hbar} \qquad k = \frac{n\pi}{L},$$

$$n_i = \sqrt{N_c N_v}\; e^{\frac{-E_g}{2kT}} \qquad \mathcal{E} = -\frac{d\psi}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

$$\psi(x,y,z) = \psi_x(x)\,\psi_y(y)\,\psi_z(z)$$

EE2 Midterm Solutions.

2/8/07

1a) A) $1\ 1\ \infty \rightarrow (110)$

B) $1\ 1\ 1 \longrightarrow (111)$

C) $1\ 1\ {-1} \rightarrow (11\bar{1})$

D) $1\ \frac{1}{2}\ 1 \rightarrow (121)$

$\infty\ 1\ \frac{1}{2} \rightarrow (012)$

1b) $\psi_I \rightarrow n=2 \qquad E \propto n^2 = 4 \qquad E_1 = 1eV$

$\psi_{II} \rightarrow n=3 \qquad E \propto 9 \qquad E_2 = 9eV$

lowest energy = $E_1 = 1eV$.

1c) $E(K) = \dfrac{\hbar^2 K^2}{2m_0} - AK^4$

$V_g = \dfrac{1}{\hbar}\dfrac{\partial E}{\partial k} \Rightarrow \dfrac{\hbar^2 K}{m_0} - 4AK^3 = 0$

$\dfrac{\hbar^2}{m_0} - 4AK^2 = 0 \qquad K = \dfrac{\pi}{a}$

$\dfrac{\hbar^2}{m_0} - \dfrac{4A\pi^2}{a^2} = 0. \qquad A = \dfrac{a^2 \hbar^2}{m_0 4\pi^2}$

$m^* = \dfrac{\hbar^2}{\frac{\partial^2 E}{\partial K^2}} \Rightarrow \qquad \dfrac{\partial^2 E}{\partial K^2} = \dfrac{\hbar^2}{m_0} - 12AK^2$

$m^*_{K=0} \Rightarrow \dfrac{\hbar^2}{\hbar^2/m_0} = M_0, \quad m^*_{K=\frac{\pi}{a}} = \dfrac{\hbar^2}{\frac{\hbar^2}{m_0} - \frac{3\hbar^2}{m_0}} = -\dfrac{M_0}{2}$

1c cont.

$$\frac{\partial V_g}{\partial k} = \frac{1}{\hbar}\left[\frac{\hbar^2}{m_0} - 12\left(\frac{\hbar^2 a^2}{4a^2 m_0}\right)k^2\right] = 0$$

$$\frac{\hbar^2}{m_0} = 12\left(\frac{\hbar^2 a^2}{4\pi^2 m_0}\right)k^2$$

$$k = \sqrt{\frac{\pi^2}{(3a^2)}} = -\frac{1}{\sqrt{3}}\left(\frac{\pi}{a}\right)$$

2a) $$p = N_v e^{-\frac{(E_F - E_V)}{KT}} \qquad p_i = N_v e^{-\frac{(E_i - E_V)}{KT}}$$

$$\frac{p}{p_i} = \frac{e^{-\frac{(E_F - E_V)}{KT}}}{e^{-\frac{(E_i - E_V)}{KT}}} = e^{-\frac{(E_F - E_i)}{KT}}$$

$$(E_F - E_i) = -KT \ln \frac{p}{p_i} \qquad p \propto N_A$$

As N_A increases the separation between E_F and E_i goes as KT times the of N_A

2b) $$p - n + N_D - N_A = 0$$

$$\frac{n_i^2}{n} - n + N_D - N_A = 0.$$

$$n^2 + n(N_A - N_D) - n_i^2 = 0$$

$$n = \frac{(N_D - N_A) \pm \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2}$$

Compensated $N_D - N_A > n_i$

$$n \approx N_D - N_A \qquad p = \frac{n_i^2}{n} \approx \frac{n_i^2}{N_D - N_A}$$

2c) $S_i \rightarrow 4e^-$ in other shell

$As \rightarrow 5e^-$

As has an extra electron $\rightarrow$ donor.

$Ga \rightarrow 3e \rightarrow$ has a lack of an electron.

$\Rightarrow$ acceptor.

$S_i \rightarrow$ in GaAs $\rightarrow$ in place of Ga $\Rightarrow$ donor, in place of As $\Rightarrow$ acceptor.

$$E = \frac{m_0 q^4}{2(4\pi\epsilon_0 \hbar n)^2} = -\frac{13.6\text{ eV}}{n^2}$$

take $n=1$ $E = -13.6\,\text{eV}$ for atom

for donor $m \rightarrow .8 m_e$, $\epsilon \rightarrow A$

so $E_D = -13.6 \frac{(.8)}{A^2}\,\text{eV}$

3a) $$\langle p \rangle = \frac{\int_0^\infty z(p) f(E) p \cdot dp}{\int_0^\infty z(p) f(E) dp}$$

$$= \frac{\int_0^{p_F} \frac{4L}{h} p\, dp}{\int_0^{p_F} \frac{4L}{h} dp} \Rightarrow \frac{\frac{p_F^2}{2}}{p_F} = \frac{1}{2} p_F$$

$$\langle p^2 \rangle = \frac{\int_0^{p_F} \frac{4L}{h} p^2 \, dp}{\int_0^{p_F} \frac{4L}{h} \, dp} = \frac{\frac{1}{3} p_F^3}{p_F} = \frac{1}{3} p_F^2$$

$$\langle E \rangle = \frac{\frac{1}{3} p_F^2}{2m} = \frac{1}{3} E_F$$

3b) $E = \frac{p^2}{2m^*}$ $\quad \sqrt{2m^* E} = p$

$$Z(p)\,dp = \frac{4L\,dp}{h}$$

$$dp = \frac{1}{2}\sqrt{2m^*}\, E^{-1/2}\, dE$$

$$Z(E)\,dE = \frac{4L}{h}\left(\frac{1}{2}\right)\left(2m^* E^{-1/2} dE\right)$$

$$= \frac{2L}{h}\sqrt{2m}\; E^{-1/2}\, dE$$

$$n = \int_0^{\infty} f(E) Z(E)\, dE = \int_0^{E_F} \frac{2L\sqrt{2m}}{h} E^{-1/2}\, dE$$

$$n = \frac{2L\sqrt{2m^*}}{h}\left(2E^{1/2}\right)\Big|_0^{E_F} = \frac{4L\sqrt{2m^*}}{h} E_F^{1/2}$$

$$E_F = \frac{n^2 h^2}{32 L^2 m^*} \qquad m'^* = \frac{1}{2} m^*$$

$$E_F' = 2E_F \qquad \text{Fermi Level doubles.}$$

3c)

A) Shows formation of solid Si from atoms. This tight binding approach shows that Band gaps develop. All of the electrons are in the valence, lower, state at low temperature. It is shown as a function of distance between the atoms.

B) This is an atom of Si –

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \quad \text{where } n = 2.$$

There are 4 s and p electrons.

C) This shows the photo electric effect with a photon incident on a material. It has a dependence

$h\nu = \phi + eV$ where ϕ is the work function. When $h\nu$ is less than ϕ no electrons are emitted.

D) The ln of n as a function of 1/T. At the highest temp. $1/T \to 0$ the material is intrinsic. Between T_1 and T_2 the material is extrinsic determined by donors & acceptors. as $T \to$ the material is into freeze out $n \propto e^{-(E_c - E_d)/2kT}$.