ECE C143A/C243A, Spring 2018 **Midterm**

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UCLA True Bruin academic integrity principles apply.
Open: Two pages of cheat sheet allowed.
Closed: Book, computer, Internet.
2:00-3:50pm.
Monday, 07 May 2018.

State your assumptions and reasoning.
No credit without reasoning.
Show all work on these pages.

Name: ____________________

Signature: ____________________

ID#: _________________

Problem 1	_____	/	30
Problem 2	_____	/	20
Problem 3	_____	/	25
Problem 4	_____	/	25
BONUS 5	_____	/	5 bonus points
Total	_____	/	105 points

1. (30 points) An alien neuron. Consider an alien species whose neurons are permeable to just two ion species: Na^+ and Li^+. In addition to resting channels, this alien species also has light-gated ion channels. When exposed to green light, Na^+ channels selectively open; when exposed to red light, Li^+ channels selectively open. At rest (equilibrium), the ion concentrations are:

Ionic Species	Intracellular concentration	Extracellular concentration
Na^+	100 mM	10 mM
Li^+	10 mM	100 mM

(a) (5 points) Calculate the resting equilibrium potential for Na^+ as well as for Li^+. Assume the alien's neurons operate at room temperature (25°) so that RT/F is 25 mV. The equation for resting equilibrium potential for ion X is:

$$\begin{aligned} E_X &= \frac{RT}{zF} \ln \frac{[X]_o}{[X]_i} \\ &= \frac{25}{z} \ln \frac{[X]_o}{[X]_i} \\ &= \frac{57.6}{z} \log_{10} \frac{[X]_o}{[X]_i} \end{aligned}$$

where as in class, z is valence, $[X]_o$ is the extracellular concentration, and $[X]_i$ is the intracellular concentration.

(b) (5 points) You record the resting membrane potential of this alien neuron to be -40 mV. Based on this information, are there more Na^+ or Li^+ ion channels open at rest? (Justify your answer in no more than 3 sentences.)

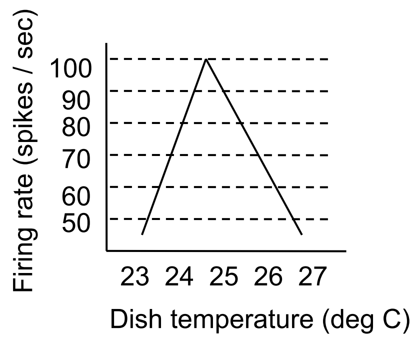
(c) (5 points) Imagine I exposed the alien neuron to red light. Does Li^+ flow into or out of the cell? Does this increase or decrease the membrane potential? (Justify your answer in no more than 4 sentences.)

(d) (5 points) Your colleague finds that shining red light, immediately followed by green light, causes the alien neuron to fire action potentials. Using the information given in this question, provide an upper bound on the action potential amplitude. Justify your calculations.

(e) (5 points) Your colleague, intrigued by the alien neuron's responsiveness to light, shines a blue light on the alien neuron. She determines that when she shines a blue light, the ion channels no longer have a refractory period. She combines this with other stimulation to make the neuron fire at 800 Hz. She says "This raises the capacity of these neurons, enabling greater computations." You think for a bit, and realize that this may not be a great idea because removing the refractory can have negative consequences. What is a negative consequence of removing the refractory period?

(f) (5 points) Finally, your colleague finds that when she exposes the neuron to infrared radiation, the neuron's cell wall inflates, so that its thickness doubles. Does this increase or decrease the speed at which an action potential is generated? How about the distance over which the action potential propagates along the alien neuron's axon? (Justify your answer in no more than 4 sentences.)

2. (20 points) Tuning curve. It's your first day in the lab, and you've just measured your first tuning curve from a worm. The tuning curve you measured is plotted below, and expresses the average action potential firing rate versus the temperature of the dish on which your worm is sitting.



(a) (8 points) You begin to ponder the use of this worm as an autonomous / mobile thermometer, whereby you could let it loose and wirelessly telemeter back the firing rate and use this to measure the temperature of the earth (i.e., cooler dirt cools the worm, and vice versa). What about this living-bio-sensor-paradigm limits your ability to precisely measure the ambient (earth's) temperature (assume earth's temperature is in 23-27 C range)? Discuss all possible limitations in this approach. (**Hint**: there are two major limitations; further recall that tuning curves describe the relationship between *averaged* firing rates and the variable of interest, which in this case is temperature.)

(b) (7 points) Daydreaming further, you envision letting several (N) identical but independent telemetry-enabled worms loose in the same region with the hope that you could use these N measurements from the same area (i.e., assume all worms in earth of same temperature) to better estimate temperature. These identical worms have identical tuning curves.

i. (4 points) Does this help to arrive at more accurate temperatures? (Justify in no more than 2 sentences.)

ii. (3 points) Assume N is arbitrarily large. Does this allow you to estimate the temperature exactly? (Justify in no more than 2 sentences.)

(c) (5 points) Finally, imagine you could change anything about the worms and their firing rates. Is there something you would change for each of the N worms from part b) above? Why? (2-3 sentences sufficient.)

3. (25 points) Poisson distribution. The number of spikes recorded per minute by an electrode is a Poisson random variable with mean 3.

   (a) (5 points) Find the probability that no spikes are recorded in a given one minute period. You don't have to evaluate exponentials (i.e., you may leave a term, like e^1 in your final expression; this applies for the rest of the exam).

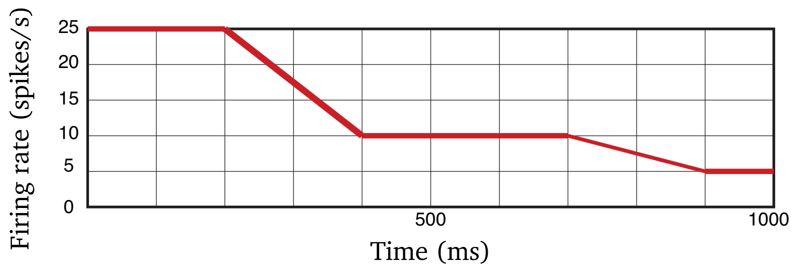
(b) (10 points) Find the probability that at least two spikes are recorded in a given 2-minute period.

(c) (10 points) Suppose that in a given minute 3 spikes were recorded. What is the conditional probability that all 3 spikes were recorded in the first 30 seconds?

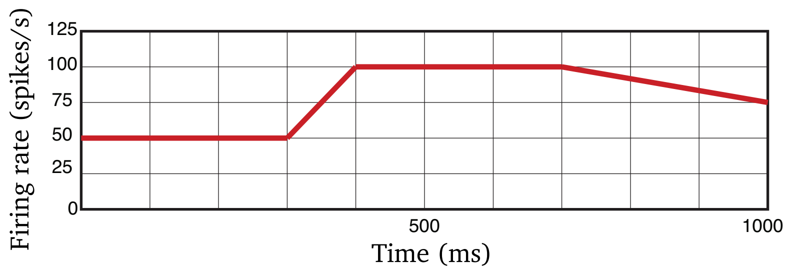
4. (25 points) Poisson processes. Assume two neurons, where neuron 1 has firing rate $y_1(t)$ and neuron 2 has firing rate $y_2(t)$. Both neurons are perfectly modeled by a Poisson process.

   (a) (5 points) To quantify the statistics of both neurons, we had an animal perform 500 identical trials and calculated firing rate through time by averaging across all trials. The across-trial average firing rate over the course of the trial is shown below.

Neuron 1, $y_1(t)$

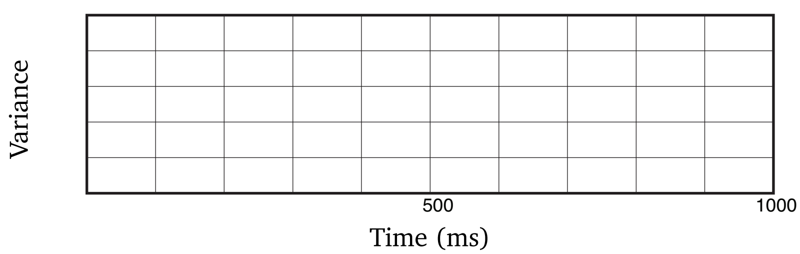


Neuron 2, $y_2(t)$

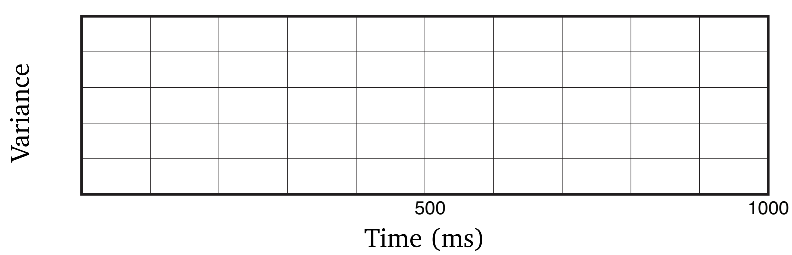


Plot the instantaneous across-trial variance for both neurons on the plots below. (There is space on the next page calculations if need be, but they are not required.)

Neuron 1



Neuron 2



(a cont.)

(b) (5 points) Assume neurons 1 and 2 are independent. Imagine that one electrode was recording neuron 1 and neuron 2 at the same time. What would be the expected number of spikes recorded between 0 and 200 ms? You may leave your answer in terms of the multplication of numbers if you don't have a calculator.

(c) (5 points) What is the probability that neuron 1 fires 1 spike in the 200 to 400 ms interval?

(d) (5 points) Consider neuron 1's spike interarrival times. Are they exponential? Are they indepedent? Justify your answer.

(e) (5 points) Imagine a decoder which weights the noisy empirically recorded firing rates of neuron 1 and 2, given by $\tilde{y}_1(t)$ and $\tilde{y}_2(t)$ respectively, to decode a variable of interest, $x(t)$. The decoder is built so that the weight of noisier neurons is reduced (indicating that the neuron is less reliable). Assume that the weights are positive ($a > 0$ and $b > 0$) and that the decoding equation is

$$x(t) = a\tilde{y}_1(t) + b\tilde{y}_2(t).$$

At time $t = 500$ ms, we estimate the firing rates (through binning) of neuron 1 and neuron 2 to be:

$$\begin{aligned} \tilde{y}_1(500) &= 10 \text{ spikes/s} \\ \tilde{y}_2(500) &= 95 \text{ spikes/s.} \end{aligned}$$

At time $t = 550$ ms, we estimate the firing rate of neuron 1 and neuron 2 to be:

$$\begin{aligned} \tilde{y}_1(550) &= 15 \text{ spikes/s} \\ \tilde{y}_2(550) &= 100 \text{ spikes/s.} \end{aligned}$$

Consider the change in $x(t)$ over this interval, i.e., $\Delta x = x(550) - x(500)$. Is the change in $x(t)$ due to neuron 1 (i) the same as, (ii) less than, or (iii) more than that due to neuron 2? Why? Justify your answer to receive full credit.

5. (5 points) Bonus Question

Assume that the number of spikes recorded by an electrode is a Poisson process with rate λ spikes/second. Also, let's denote the arrival times of the spikes as $T_1, T_2, T_3, \cdots$. We define the random variable W in the following manner,

$$W = R - T_1 + R - T_2 + R - T_3 + R - T_4 + \cdots$$

where R is some deterministic time. Find $\mathbb{E}[W]$. (Hint: You may find the following property useful: $\mathbb{E}[W] = \sum_{k=0}^{\infty} \mathbb{E}[W|N(R) = k]\mathbb{P}(N(R) = k)$. This property, which you can use without proof, is known as the law of total expectation, aka iterated expectation, aka the tower rule.)