Solution to Quiz 2

1 Energy stored in the system is the total work done

$$W_{total} = W_1 + W_2 + W_3$$

$$= 0 + Q_2 V_{21} + Q_3 [V_{31} + V_{32}]$$

$$= 0 + Q_2 \frac{Q_1}{4\pi\varepsilon_0 |(0,0,1) - (0,0,0)|} + \frac{Q_3}{4\pi\varepsilon_0} \left[\frac{Q_1}{|(1,0,0) - (0,0,0)|} + \frac{Q_2}{|(1,0,0) - (0,0,1)|} \right]$$

$$= \frac{1}{4\pi\varepsilon_0} \left(Q_1 Q_2 + Q_1 Q_3 + \frac{Q_2 Q_3}{\sqrt{2}} \right)$$

$$= \frac{1}{4\pi \times 8.85 \times 10^{-12}} \left(-4 - 3 + \frac{12}{\sqrt{2}} \right) \times 10^{-18}$$

$$= 9 \times 10^{-9} \left(\frac{12}{\sqrt{2}} - 7 \right) \simeq 13.4 \text{ nJ}.$$

alternatively

$$W_{total} = \frac{1}{2} \sum_{m=1}^{3} Q_m V_m = \frac{1}{2} \left(Q_1 V_1 + Q_2 V_2 + Q_3 V_3 \right)$$

$$= \frac{Q_1}{2} \left[\frac{Q_2}{4\pi\varepsilon_0 (1)} + \frac{Q_3}{4\pi\varepsilon_0 (1)} \right] + \frac{Q_2}{2} \left[\frac{Q_1}{4\pi\varepsilon_0 (1)} + \frac{Q_3}{4\pi\varepsilon_0 (\sqrt{2})} \right]$$

$$= 13.4 \text{ nJ} \text{ (also)} + \frac{Q_3}{2} \left[\frac{Q_1}{4\pi\varepsilon_0 (1)} + \frac{Q_2}{4\pi\varepsilon_0 \sqrt{2}} \right] = \frac{1}{4\pi\varepsilon_0} \left(Q_1 Q_2 + Q_1 Q_3 + \frac{Q_2 Q_3}{\sqrt{2}} \right)$$

2 $n_e = 10^{29}\ m^{-3}$

$\rho_v = e n_e = (10^{29})\cdot(1.6\times10^{-19}) = -1.6\times10^{10}\ C/m^3$

a) $J = \sigma E = (5\times10^{7})(10\times10^{-3}) = 500\ kA/m^2$

b) $I = JS = (5\times10^{5})\left(\frac{\pi d^2}{4}\right) = 5\times10^{5}\times\frac{\pi\times(10^{-3})^2}{4} = 0.4\ A$

c) $J = \rho_v v_{drift} =$

$v_{drift} = J/\rho_v = -\frac{5\times10^{5}}{1.6\times10^{10}} = -3.125\times10^{-5}\ m/s.$

3 For a uniform conductor

$R = \ell/\sigma S =$

$S = d^2 - \pi r^2 = 3^2 - \pi\left(1/2\right)^2 = \left(9-\frac{\pi}{4}\right) cm^2$

$R = \frac{4}{5\times10^{7}\left[\left(9-\pi/4\right)\times10^{-4}\right]} = 97.4\ \mu\Omega.$