EE 1, QUIZ 1 50

Name: Solutions

ID #

12:30 - 1:15 pm

15

~~14~~ 1 Write expressions for the following:

1 a) Coulombs law $\vec{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^2}\vec{a}_R$ (N)

1 b) Electric field intensity $\vec{E} = \frac{Q}{4\pi\varepsilon_o R^2}\vec{a}_R$ (V/m)

1 c) Gauss' law $\oint_S \vec{D}\cdot d\vec{s} = Q_{enclosed}$

1 d) Divergence theorem $\oint_S \vec{D}\cdot d\vec{s} = \int_V \nabla\cdot\vec{D}\,dv$

1 5) Electric field intensity in terms of scalar potential $\vec{E} = -\nabla V$

2 6) surface elements (side and top) on a cylinder (vector)

$d\vec{s} = \rho\, d\phi\, dz\, \vec{a}_\rho$ (side)

$d\vec{s} = \rho\, d\rho\, d\phi$ (top)

1 7) volume element on a cylinder

$dV = \rho\, d\rho\, d\phi\, dz$

3 8) Calculate $\nabla \cdot \nabla \times A$ in rectangular co-ordinates

$\nabla\cdot\nabla\times\vec{A} \equiv 0$

2 9) Calculate $\nabla \times \nabla V$ in rectangular co-ordinates

$\nabla\times\nabla V \equiv 0$

2 10) Calculate $\nabla\cdot\nabla V$ in rectangular co-ordinates.

$\nabla\cdot\nabla V = \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$.

(10) 2 The electric ~~field~~ flux density is specified as

$$\underline{D}(x,y,z) = \varepsilon_0 \left[24\,xy\; a_x + 12\,(x^2+2)\, a_y + 18\, z^2\, a_z \right] \; C/m^2$$

$\varepsilon_0 = (1/36\pi) \times 10^{-9}$ F/m

a) Find $\underline{D}$ and $\underline{E}$ at $P(4,5,2)$

(1) $$\vec{E} = \vec{D}/\varepsilon_0 = 24xy\,\vec{a}_x + 12(x^2+2)\vec{a}_y + 18z^2\,\vec{a}_z \quad (V/m)$$

(1) At $P = \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix}$, $\vec{E} = 480\,\vec{a}_x + 216\,\vec{a}_y + 72\,\vec{a}_z \quad (V/m)$

(1) $$\vec{D} = \varepsilon_0 \vec{E} = \frac{1}{36\pi} \times 10^{-9}\, \vec{E}(P)$$

$$= 8.854 \times 10^{-12} \times (480\vec{a}_x + 216\vec{a}_y + 72\vec{a}_z) \; (C/m^2)$$

b) The scalar component of E at $P(4,5,2)$ in the direction of $\boldsymbol{A}_N = (2a_x + 6a_y + 2a_z)$

(1) Unit vector $\vec{a}_N = \dfrac{2\vec{a}_x + 6\vec{a}_y + 2\vec{a}_z}{\sqrt{44}}$

(2) $$E_{a_N} = \vec{E}\cdot\vec{a}_N = (480\vec{a}_x + 216\vec{a}_y + 72\vec{a}_z)\cdot \frac{2\vec{a}_x + 6\vec{a}_y + 2\vec{a}_z}{\sqrt{44}}$$

$$= \frac{1}{\sqrt{44}}(960 + 1296 + 144)$$

(1) $$= 361.81 \; (V/m)$$

c) The vector component of $\underline{E}$ at $P(4,5,2)$ in the direction of $\boldsymbol{A}_N$.

(2) $$\vec{E}_{a_N} = E_{a_N}\vec{a}_N = 361.81 \times \frac{2\vec{a}_x + 6\vec{a}_y + 2\vec{a}_z}{\sqrt{44}}$$

(1) $$= 109.09\,\vec{a}_x + 327.27\,\vec{a}_y + 109.09\,\vec{a}_z \; (V/m)$$

(5) 3

How much charge is contained inside a cylinder bounded by $0 \leq \rho \leq 1$ m and $0 \leq z \leq 2$ m if the charge density $\rho_v = 5\rho^3$ C/m^3 exists ?

(3)

$$Q_{enclosed} = \int_V \rho_v \, dV = \int_0^2 \int_0^{2\pi} \int_0^1 5\rho^3 \, \rho \, d\rho \, d\phi \, dz$$

$$= \int_0^2 dz \int_0^{2\pi} d\phi \int_0^1 5\rho^4 \, d\rho$$

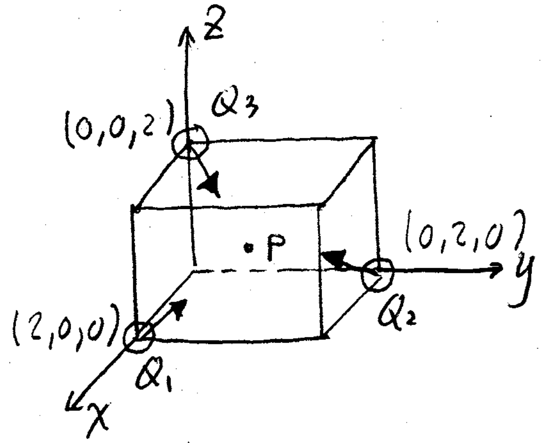
(1)

$$= 2 \times 2\pi \times \rho^5 \Big|_0^1$$

(1)

$$= 4\pi \quad (C)$$

(10)



If three identical charges of 3 nC each were placed at the corners of a cube as shown, what will be the electric field at the center point P ?

② The coordinates of the center of the cube P are $(1, 1, 1)$.

③ $$\vec{R}_{1p} = \begin{pmatrix}1\\1\\1\end{pmatrix} - \begin{pmatrix}2\\0\\0\end{pmatrix} = \begin{pmatrix}-1\\1\\1\end{pmatrix}, \quad R_{1p} = \sqrt{3}$$

$$\vec{R}_{2p} = \begin{pmatrix}1\\1\\1\end{pmatrix} - \begin{pmatrix}0\\2\\0\end{pmatrix} = \begin{pmatrix}1\\-1\\1\end{pmatrix}, \quad R_{2p} = \sqrt{3}$$

$$\vec{R}_{3p} = \begin{pmatrix}1\\1\\1\end{pmatrix} - \begin{pmatrix}0\\0\\2\end{pmatrix} - \begin{pmatrix}1\\1\\-1\end{pmatrix} \quad R_{3p} = \sqrt{3}$$

By principle of linear superposition,

③ $$\vec{E}(p) = \sum_{m=1}^{3} \frac{Q_m}{4\pi \varepsilon_0 R_{mp}^3} \vec{R}_{mp}$$

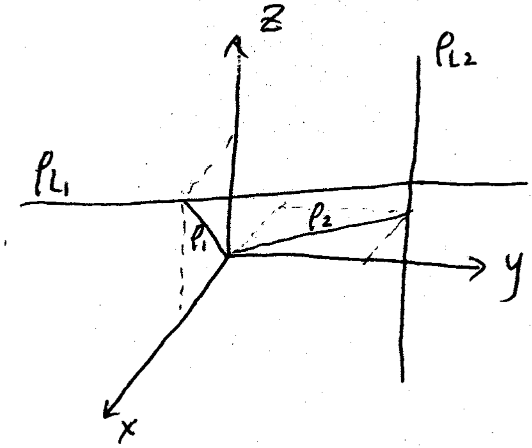
$$= \frac{3\times10^{-9}}{4\pi \times 8.854\times10^{-12} \times (\sqrt{3})^3} \times \left[\begin{pmatrix}-1\\1\\1\end{pmatrix} + \begin{pmatrix}1\\-1\\1\end{pmatrix} + \begin{pmatrix}1\\1\\-1\end{pmatrix} \right]$$

② $$= 5.19 \begin{pmatrix}1\\1\\1\end{pmatrix} = 5.19(\vec{a}_x + \vec{a}_y + \vec{a}_z) \quad (\mathrm{V/m})$$

(10) 4

Two uniform line charges, 4 nC/m each are located at $x=1$, $z=2$ and at $x=-1$ and $y=2$ in free space. If the potential at the origin is 100 V find V at P(4, 1, 3).

(1) $\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\vec{a}_\rho$.



(2) $$V = -\int \vec{E}\cdot d\rho\,\vec{a}_\rho + C$$
$$= -\int \frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a}_\rho \cdot d\rho\,\vec{a}_\rho + C$$
$$= -\frac{\rho_L}{2\pi\varepsilon_0}\ln\rho + C.$$

(1) The total potential field at any point is
$$V = V_1 + V_2 = -\frac{\rho_L}{2\pi\varepsilon_0}(\ln\rho_1 + \ln\rho_2) + C \quad (V).$$

(2) At the origin (0, 0, 0), $\rho_1 = \sqrt{1^2+2^2} = \sqrt{5}$,
$\rho_2 = \sqrt{(-1)^2+2^2} = \sqrt{5}$.

(1) $$V(0) = -\frac{4\times10^{-9}}{2\pi\times8.854\times10^{-12}}\times(\ln\sqrt{5}+\ln\sqrt{5}) + C = 100$$
$$\Rightarrow C = 215.72.$$

Therefore, $V = -\frac{\rho_L}{2\pi\varepsilon_0}(\ln\rho_1 + \ln\rho_2) + 215.72$. (V).

(2) At P(4, 1, 3), $\rho_1 = \sqrt{(4-1)^2+0^2+(3-2)^2} = \sqrt{10}$.
$\rho_2 = \sqrt{(4+1)^2+(1-2)^2+0^2} = \sqrt{26}$.

(1) $$V(P) = -\frac{4\times10^{-9}}{2\pi\times8.854\times10^{-12}}\times(\ln\sqrt{10}+\ln\sqrt{26}) + 215.72$$
$$= 15.81 \quad (V).$$