EE 1 QUIZ II Name: Solutions

45 mins CLOSE BOOK ID #:

15 + 2

1 a Using Gauss' law as a starting point, find an expression for capacitance of a sphere radius a. ($\varepsilon = \varepsilon_0 = 8.85\times10^{-12}$ F/m)

(1) Gauss's Law: $\oint_S \vec{D}\cdot d\vec{S} = Q_{encl}$

(1) Because of symmetry, Left $= \oint_S \varepsilon E_r \vec{a}_r \cdot ds\,\vec{a}_r = \varepsilon E_r \oint_S ds = \varepsilon E_r \cdot 4\pi r^2$

Right $= \int_S \rho_s ds = \rho_s \cdot 4\pi a^2$.

(1) $\Rightarrow E_r = \rho_s 4\pi a^2 / 4\pi\varepsilon r^2$.

(2) $V = -\int_\infty^r \vec{E}\cdot d\vec{L} = -\int_\infty^r \rho_s a^2/\varepsilon r^2 \cdot \vec{a}_r \cdot dr\,\vec{a}_r = \frac{\rho_s a^2}{\varepsilon r}$, $V_a = \frac{\rho_s a}{\varepsilon}$

(1) $C = Q/V_a = \rho_s \cdot 4\pi a^2 / \frac{\rho_s a}{\varepsilon} = 4\pi\varepsilon a$.

1 b Considering the earth as a conducting sphere of radius $a = 6.37\times10^6$ m, determine it's capacitance.

(2) $$C_{earth} = 4\pi \times 8.85\times10^{-12} \times 6.37\times10^6 = 7.08\times10^{-4} \text{ (F)}.$$

1 c If the air at sea level has an electrical breakdown threshold of 30 kV/cm, what is the maximum charge that can exist on the surface of the earth?

If $E_r = E_{breakdown}$, air will become a plasma.

(2) $Q_{max} = \oint_S \vec{D}\cdot d\vec{S} = \varepsilon E_r \cdot 4\pi a^2$

(1) $= 8.85\times10^{-12} \times 30\times10^3/10^{-2} \times 4\pi \times (6.37\times10^6)^2$

(1) $= 1.35\times10^{10}$ (C)

Problem 1 (continued)

1 d What will be the potential at a at this point of breakdown?

1

$$V_{max} = \frac{Q_{max}}{4\pi\varepsilon_0 a}$$

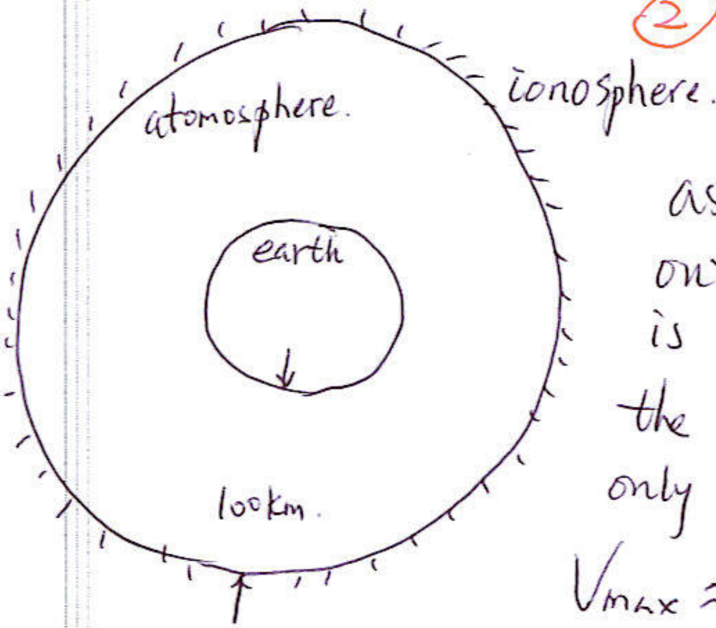
$$= \frac{1.35\times10^{10}}{4\pi\times8.85\times10^{-12}\times6.37\times10^{6}}$$

2

$$= 1.91\times10^{13}\ (V)$$

1 e Is this answer for the potential reasonable? Why or why not? (bonus)

2



Not reasonable.

This value of the potential assumes that the radius of the outer concentric conducting sphere is infinitely far away. In fact, the ionosphere above the earth is only 100 km away.

$$V_{max} \approx \frac{Q_{max}}{4\pi\varepsilon}\left(\frac{1}{a}-\frac{1}{b}\right)$$

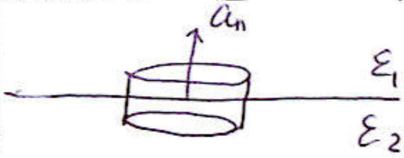
$a = 6.37\times10^{6}$ m.

$b = 6.37\times10^{6} + 100\times10^{3} = 6.47\times10^{6}$ m

$$V_{max} = 2.95\times10^{11}\ (V)$$

10

2 a Using $-\oint_L \underline{E}\cdot d\underline{L} = 0$ and $\oint \underline{D}\cdot d\underline{S} = Q_{encl}$ find the boundary conditions for the tangential and normal components of the electric field (E_t and E_n) at the interface of two dielectrics with permittivities (or dielectric constants) ε_1 and ε_2



$$\oint_S \vec{D}\cdot d\vec{S} = Q \Rightarrow D_{n1}\cdot A - D_{n2}\cdot A = \rho_s \cdot A$$

$$\Rightarrow D_{n1} - D_{n2} = \rho_s$$

No charge at the interface of two dielectrics, $\rho_s = 0$

$$\Rightarrow D_{n1} = D_{n2} \quad \Rightarrow \varepsilon_1 E_{n1} = \varepsilon_2 E_{n2}$$

$$\oint_c \vec{E}\cdot d\vec{L} = 0 \Rightarrow E_{t1}\cdot L - E_{t2}\cdot L = 0$$

$$\Rightarrow E_{t1} = E_{t2}$$

2b An electric field of 10 kV/m is applied across a mylar dielectric with a relative permittivity of 5.6. What is the electric field inside the mylar ?

Across the interface, E-field is in the normal direction.

$$D_{n1} = D_{n2} \Rightarrow \varepsilon_0 \cdot E_1 = \varepsilon_0 \varepsilon_r E_2$$

$$\Rightarrow E_2 = \frac{E_1}{\varepsilon_r} = \frac{10\,k}{5.6} = 1.79\ kV/m$$

2c What is the polarization inside the mylar?

$\cancel{D_2 = \varepsilon_0 E_2 + P_2}$

$\cancel{P_2 = \varepsilon_0}$

$$\varepsilon_r = 1 + \chi_e \Rightarrow \chi_e = \varepsilon_r - 1 = 5.6 - 1 = 4.6$$

$$P_2 = \chi_e \varepsilon_0 E_2 = 4.6 \times 8.85\times10^{-12} \times 1.79\times10^{3}$$

$$= 7.29 \times 10^{-8}\ (C/m^2)$$

(10)

3 An infinitely long wire carries a 15 A current and lies along the z axis. What is the magnetic field intensity $\underline{H}$ at a point $P(\sqrt{20}, 0, 4)$ in rectangular co-ordinate system?

(1) Ampere's Law : $\oint_c \vec{H} \cdot d\vec{L} = I_{encl.}$

(1) $\Rightarrow \int_0^{2\pi} H_\phi(\rho)\, \vec{a}_\phi \cdot \rho\, d\phi\, \vec{a}_\phi = 2\pi\rho H_\phi(\rho) = I$

(1) $\Rightarrow \vec{H} = \dfrac{I}{2\pi\rho} \vec{a}_\phi$

(1) At point $P = \begin{pmatrix} \sqrt{20} \\ 0 \\ 4 \end{pmatrix}$, $\phi = \tan^{-1}\left(\dfrac{0}{\sqrt{20}}\right) = 0^\circ$, $\rho = \sqrt{20 + 0^2} = \sqrt{20}$

$\vec{a}_\phi = \vec{a}_y$.

(1) $\vec{H} = \dfrac{I}{2\pi\rho} \vec{a}_\phi = \dfrac{15}{2\pi\sqrt{20}} \vec{a}_y = 0.53\, \vec{a}_y \quad (A/m)$

A differential current element $I d\underline{L} = 10\, \underline{a}_y$ lies along the y axis at (0,5,0). Using Biot-Savart law $d\underline{H} = (I d\underline{L} \times \underline{a}_R) / 4\pi R^2$ find the components of the magnetic field at $P(3,5,4)$ in the rectangular co-ordinate system.

(1) $R = \sqrt{3^2 + 0^2 + 4^2} = 5,$

(1) $\vec{a}_R = \vec{R}/R = \dfrac{3\vec{a}_x + 4\vec{a}_z}{5}$

(2) $d\vec{H} = \dfrac{I d\vec{L} \times \vec{a}_R}{4\pi R^2} = \dfrac{10\, \vec{a}_y \times (3\vec{a}_x + 4\vec{a}_z)/5}{4\pi\, 5^2}$

(1) $= \dfrac{1}{50\pi}\left(-3\vec{a}_z + 4\vec{a}_x\right) \quad (A/m)$.