Question 1: Number System Conversion

Find the value of x in the following equations. Show your work for credit.

a) $15670_8 = x_{16}$

001 | 101 | 110 | 111 | 000

0001 1011 1011 1000

$\boxed{x_{16} = 1BB8}$

b) $277_{10} = x_8$

$$\begin{array}{r} 34 \\ 8\overline{)277} \\ \underline{24} \\ 37 \\ \underline{32} \\ 5 \end{array} \qquad \begin{array}{r} 4 \\ 8\overline{)34} \\ \underline{32} \\ 2. \end{array} \qquad \begin{array}{r} 0 \\ 8\overline{)4} \\ \underline{0} \\ 4 \end{array}$$

$\boxed{x_8 = 425}$

Question 2: Universal Set

Show that the set {XNOR, OR} is a universal. You can use the constant 0.

XNOR	0	1
0	1	0
1	0	1

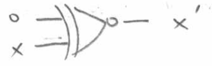
$z = x_1 x_0 + x_1' x_0'$

$x_0 = 0.$ $z = \cancel{x \cdot 0} + x' \cdot 1 = x'$

known universal set { NOT, OR }

To implement NOT

XNOR (x , 0) = x'



OR = OR.

Thus { XNOR, OR } is an universal set because it can implement a known universal set { NOT, OR }.

✓

Question 3: Boolean Algebra

a) Find the minimal representation of the following Boolean function. Show your work for credit.

f = a'+ a(a'b+b'c)'

$f = a' + a(a'b)'(b'c)'$

$= a' + a(a+b')(b+c')$

$= a' + (a + ab')(b+c')$

$= a' + ab + ac' + ab'c'$

$= a' + a(b + c' + b'c')$

$= a' + a(b + c' + c')$

$= a' + a(b + c')$

$\boxed{f = a' + b + c'}$

✓

b) Find all the minterms for the following switching expression. Show your work for credit.

F(a, b, c, d) = ((bd+b')'+ac)'

$F(a,b,c,d) = ((bd+b')' + ac)'$

$= ((bd)'(b')' + ac)'$

$= ((b'+d')(b) + ac)'$

$= (bd' + ac)'$

$= (bd')'(ac)'$

$= (b'+d)(a'+c')$

$= a'b' + b'c' + a'd + c'd$

$= a'b'(c+c') + (a+a')b'c' + a'd(b+b') + (a+a')c'd$

$a'b'$	$a'b'c$	$a'b'cd$	3
	$a'b'c'$	$a'b'cd'$	2
		$a'b'c'd$	1
		$a'b'c'd'$	0
$b'c'$	$ab'c'$	$ab'c'd$	9
	$a'b'c'$	$ab'c'd'$	8
		$a'b'c'd$	1
		$a'b'c'd'$	0
$a'd$	$a'bd$	$a'bcd$	7
	$a'b'd$	$a'bc'd$	5
		$a'b'cd$	3
		$a'b'c'd$	1
$c'd$	$ac'd$	$abc'd$	13
	$a'c'd$	$ab'c'd$	9
		$a'b'c'd$	1
		$a'bc'd$	5

= ~~a'b'c~~ + ~~ab'c'~~ + ~~ab'c'~~ + ~~[illegible]~~ + ~~a'bd'~~ + ~~a'b'd'~~ + ac'd + a'c'd

1001 0110

$= \overset{3}{a'b'cd} + \overset{2}{a'b'cd'} + \overset{1}{a'b'c'd} + \overset{0}{a'b'c'd'} + \overset{9}{ab'c'd} + \overset{8}{ab'c'd'} + \overset{6}{a'bcd'} +$

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$\overset{4}{a'bc'd'}$ + ~~a'b'cd'~~ + ~~a'b'c'd'~~ + $\overset{13}{abc'd}$ + ~~ab'c'd~~ + $\overset{5}{a'bc'd}$ + $\overset{1}{a'b'c'd}$.

$\boxed{\sum m(0,1,2,3,5,7,8,9,13)}$

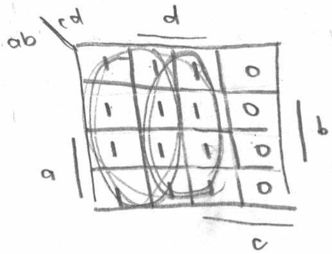
✓

Question 4: Multiplexers

Implement the f(a, b, c, d) = one-set(0, 1, 3, 4, 5, 7, 8, 9, 11, 12, 13, 15) using the smallest number of multiplexers. Note that no other gates are available, but use can use the constants 0 and 1. For credit show your work.

	a b c d	f
0	0 0 0 0	1
1	0 0 0 1	1
2	0 0 1 0	0
3	0 0 1 1	1
4	0 1 0 0	1
5	0 1 0 1	1
6	0 1 1 0	0
7	0 1 1 1	1
8	1 0 0 0	1
9	1 0 0 1	1
10	1 0 1 0	0
11	1 0 1 1	1
12	1 1 0 0	1
13	1 1 0 1	1
14	1 1 1 0	0
15	1 1 1 1	1

✓



1	1	1	0
1	1	1	0
1	1	1	0
1	1	1	0

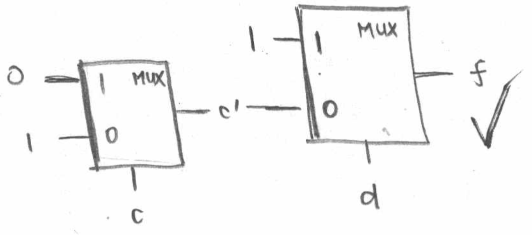
$z = c' + d$ ✓

$z = c' + d$

$= (1)d + (c')d'$

$z = (1)d + (0c + 1c')d'$

15



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$z = x_1 s + x_0 s'$

$z = 0c + 1c' = c'$.

25

Question 5: NOR-NOR network

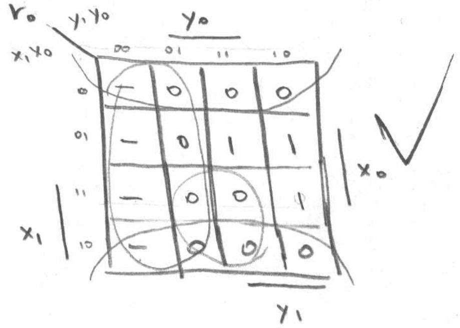
Give a minimal NOR-NOR network that implements functionality x(x1, x0) mod y(y1, y0) = r(r1, r0). For example if x=11 and y=10, the r=01. That is 3 modulo 2 is 1. Division by 0 will never be requested.

a) Provide the truth table.

$x_1 x_0$	$y_1 y_0$	$r_1 r_0$
0 0	0 0	– –
0 0	0 1	0 0
0 0	1 0	0 0
0 0	1 1	0 0
0 1	0 0	– –
0 1	0 1	0 0
0 1	1 0	0 1
0 1	1 1	0 1
1 0	0 0	– –
1 0	0 1	0 0
1 0	1 0	0 0
1 0	1 1	1 0
1 1	0 0	– –
1 1	0 1	0 0
1 1	1 0	0 1
1 1	1 1	0 0.

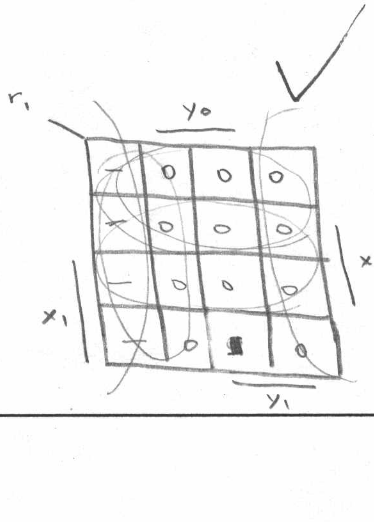
x mod y = r.

b) Provide the k-maps.



NOR - NOR.

OR - AND = implicates.



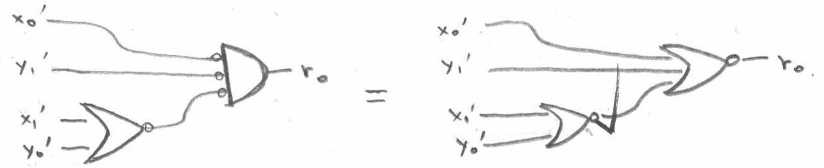
c) Provide the switching expressions.

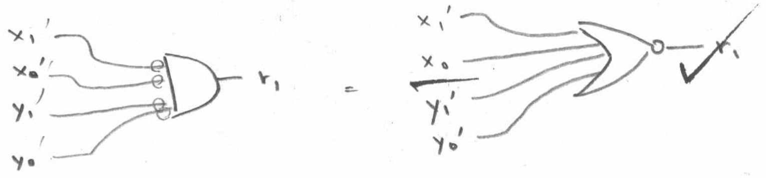
$r_0 = (x_0)(y_1)(x_1' + y_0')$

$r_1 = (x_1)(x_0')(y_1)(y_0)$

✓

d) Provide the NOR-NOR network. Assume you have the negation of the input bits available.





Question 6: State Diagram

Design a Mealy state diagram for a digital lock with the following functionality. Assume that there are two push-buttons, A and B, that are available to enter the combination, and you are guaranteed that the buttons can not be activated simultaneously.

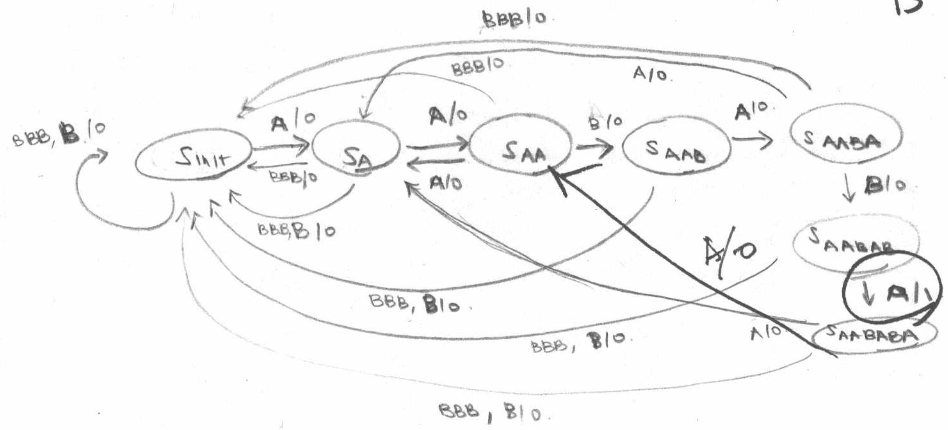
a. The combination is A-A-B-A-B-A. If this sequence is correctly entered, an output signal is asserted that causes the lock to open.
b. For any state, three B pulses in a row should reset to the initial state.

output = { 1 OPEN
0 otherwise.

BBB = 3 B pulses in a row ⇒ Sinit.

15

initial state = Sinit.



Question 7: State Table Minimization

Reduce the state table shown below and show the equivalent minimal state table. Show all the steps for credit.

Present State	State Inputs	
	x=a	x=b
A	B, 0	C, 0
B	D, 0	E, 0
C	F, 0	G, 0
D	D, 0	G, 0
E	F, 0	G, 0
F	D, 0	E, 1
G	F, 0	G, 0

Group 1: A, B, C, D E, G.

Group 2: F

	A	B	C	D	E	G	F
x=a	1	1	2	1	2	2	
x=b	1	1	1	1	1	1	

Group 1 : ABD.

Group 2: CEG

Group 3: F.

	A	B	D	C	E	G	F
x=a	1	1	1	3	3	2	
x=b	2	2	2	2	2	2.	

STOP.

MINIMIZED GROUPS:

Group 1 : A,B,D

Group 2 : C, E, G

Group 3: F.

Minimized Table.

	Input	
PS	x=a	x=b
1	1, 0	2, 0
2	3, 0	2, 0
3	1, 0	2, 1.
		NS, Output