Problem 1 (10 points)

Reduce the following expression using Boolean algebra postulates and theorems. The simplified expression should have the minimum number of gates. Show the intermediate steps.

$$f(a,b,c,d) = \overline{\overline{(acd)}(\bar a + \bar b + \bar d)(\overline{ad} + c)} + \overline{a\bar b}(\bar a + \bar b c + \bar b\bar c)$$

$$= acd + \overline{\bar a + \bar b + \bar d} + \overline{\overline{ad} + c} + (\bar a + b)(\bar a + \bar b c + \bar b \bar c)$$

$$= acd + abd + ad\bar c + \bar a + \bar a\bar b c + \bar a \bar b \bar c + \bar a b + b\bar b c + b \bar b \bar c$$

$$= a(cd + bd + \bar c d) + \bar a(1 + \bar b c + \bar b \bar c + b) + 0 + 0$$

$$= a(d(\bar c + c + b)) + \bar a$$

$$= a(d)$$

$$= \boxed{\bar a + ad}$$

✓ 6

Problem 2 (15 points)
Consider the following function

$$f(a, b, c, d) = \Sigma m(1, 7, 9, 11, 13, 15).$$

(a) (8 points) Use K-maps to minimize **both** the sum of products and products of sums forms. Write the Boolean expressions.

(b) (7 points) Implement the function using the minimal number of gates. You can use either NOR gates or NAND gates with maximum number of four inputs per gate.

(a) Sum of Products:

			c	c	
		0_{0}	1_{1}	0_{3}	0_{2}
		0_{4}	0_{5}	1_{7}	0_{6}	b
a		0_{12}	1_{13}	1_{15}	0_{14}	b
a		0_{8}	1_{9}	1_{11}	0_{10}
			d	d	

$$f(a,b,c,d) = ad + bcd + \bar{a}\bar{b}\bar{c}d$$ X

Product of Sums:

			c	c	
		0	1	0	0
		0	0	1	0	b
a		0	1	1	0	b
a		0	1	1	0
			d	d	

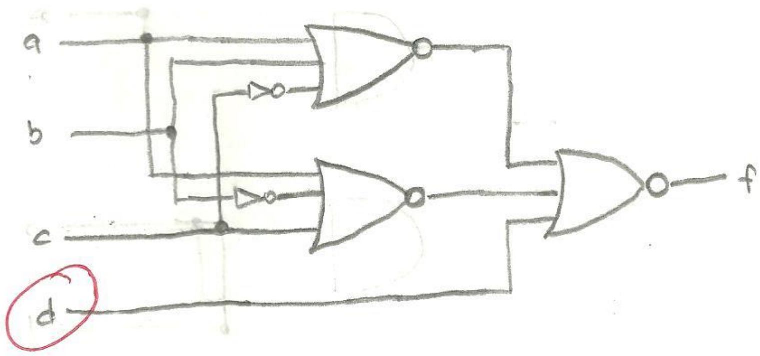
6

$$f(a,b,c,d) = d\,(a+b+\bar{c})(a+\bar{b}+c)$$ ✓

(b) See next page

NOR Gate Implementation

(b)



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4.5

Problem 3 (20 points)

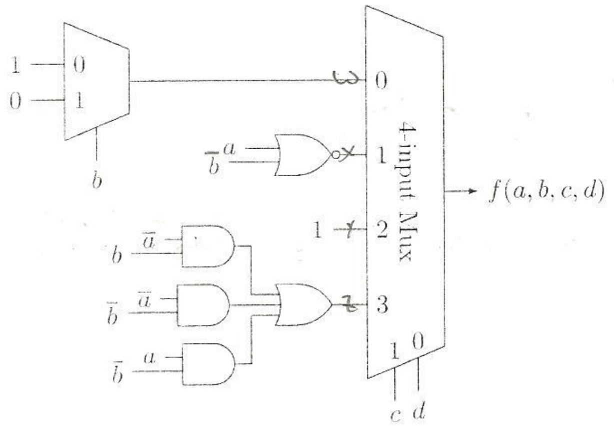
Consider the following function shown below.

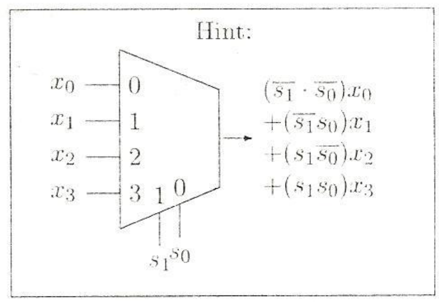
(a) (3 points) Write the switching expression for the multiplexer circuit below. The multiplexers have binary select inputs. The expression need not be simplified.

(b) (3 points) Determine the prime implicants and the essential prime implicants for this expression.

(c) (8 points) Find the minimal sum-of-product for the switching expression f.

(d) (6 points) Does this function have a unique minimal sum-of-products? If not, list any other minimal sum-of-products expressions.





(a) $(\bar c \bar d) w + (\bar c d) x + (c \bar d) y + c d z$

$w = \bar b$ $x = \overline{a + \bar b} = \bar a b$ $y = 1$, $z = \bar a b + \bar a \bar b + a \bar b = \bar a (b + \bar b) + a \bar b$

$= \bar a + a \bar b$

$= \bar a + \bar b$

$\bar b \bar c \bar d + \bar a b \bar c d + c \bar d + c d (\bar a + \bar b)$

$\boxed{\bar b \bar c \bar d + \bar a b \bar c d + c \bar d + \bar a c d + \bar b c d}$

(b)

			c	c	
	1	0	1	1	
	0	1	1	1	b
a	0	0	0	1	b
a	1	0	1	1	
		d	d		

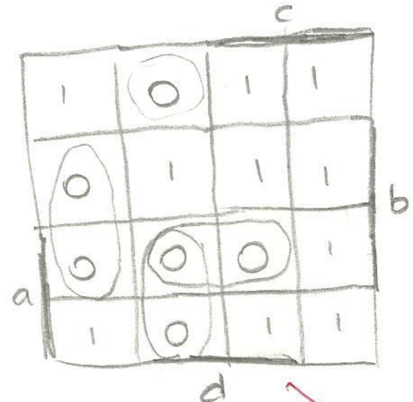
Prime Implicants: -0.5

$c \bar d$, $\bar a c$, $\bar b \bar d$, $\bar a b d$, $\cancel{a \bar b c}$

Essential PIs:

$c \bar d$, $\cancel{\bar a c}$, $\bar b \bar d$, $\bar a b d$, $\cancel{a \bar b c}$ -1

(c)



$$\cancel{\boxed{(a+b+c+\bar{d})(\bar{b}+c+d)(\bar{a}+\bar{b}+\bar{d})(\bar{a}+c+\bar{d})}}$$ POS

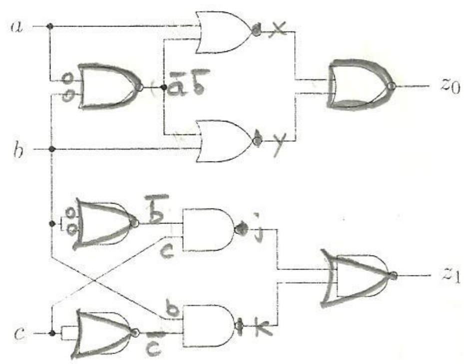
(d) Yes, there is a unique minimum sum of products.

-14

10

Problem 4 (10 points)

Analyze the NAND-NOR network shown in the figure below. Obtain switching expressions for the outputs z_0 and z_1.



$x = a + \bar{a}\bar{b} = a + \bar{b}$

$y = b + \bar{a}\bar{b} = \bar{a} + b$

$z_0 = (a + \bar{b})(\bar{a} + b)$

$a\bar{a} + \bar{a}\bar{b} + \bar{b}b + ab$

$= \bar{a}\bar{b} + ab$ ✓

$j = \bar{b}c$

$k = b\bar{c}$

$z_1 = \bar{b}c + b\bar{c}$

$$\boxed{\begin{array}{l} z_0 = \bar{a}\bar{b} + ab \\ z_1 = \bar{b}c + b\bar{c} \end{array}}$$

✓

Problem 5 (15 points)

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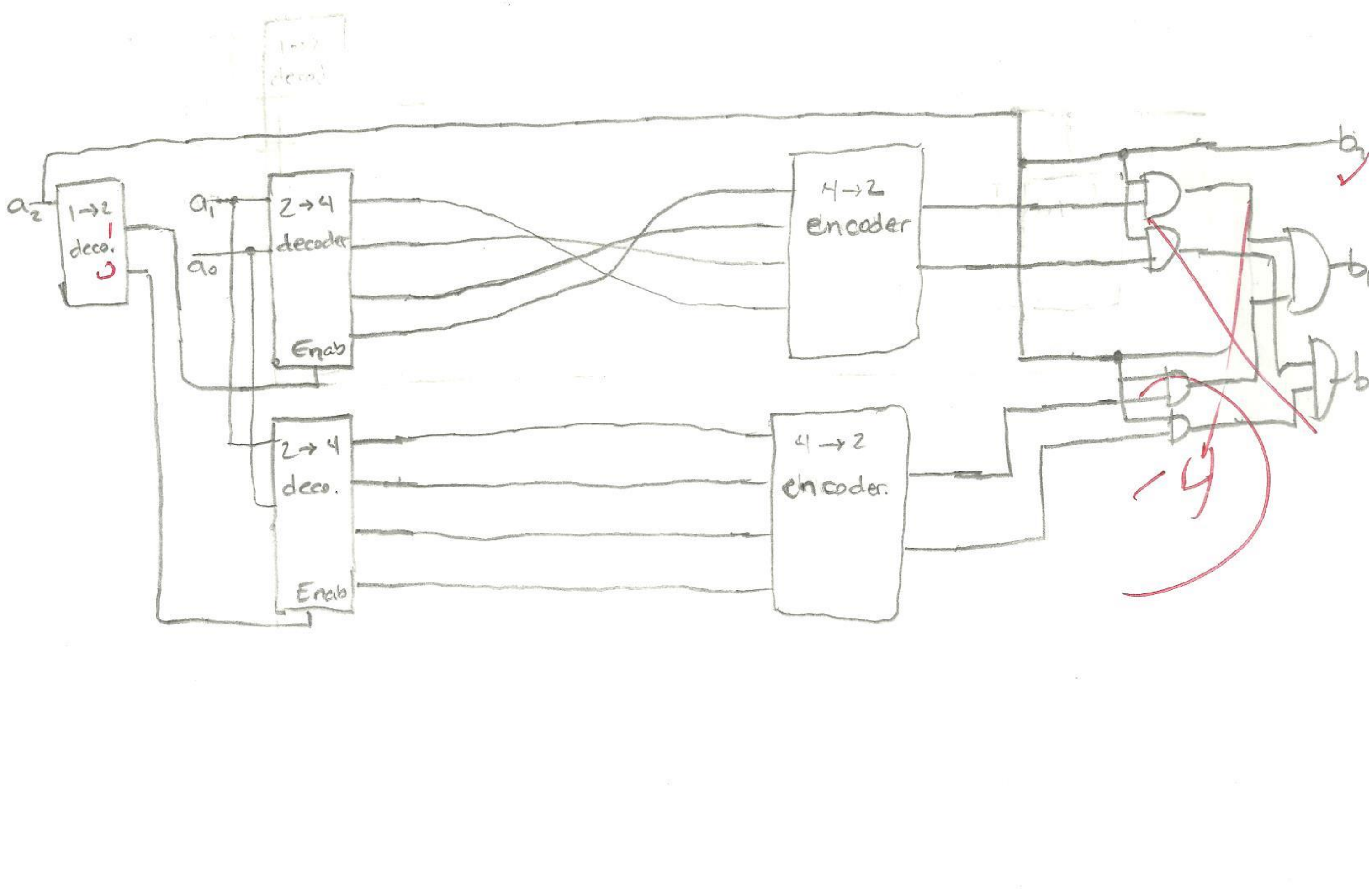
Design a combinational circuit that converts a 3-bit sign-and-magnitude number, a, into a 3-bit one's complement number, b. You are allowed to use any combination of the following blocks:

- Decoders: $1 \rightarrow 2$ and $2 \rightarrow 4$ decoders
- Encoders: $2 \rightarrow 1$ and $4 \rightarrow 2$ encoders
- Logic gates: Use either OR gates or AND gates, **but not both**

Every block or wire must be clearly labeled.

Decimal	Sign-Magnitude	One's Complement
-3	111	100
-2	110	101
-1	101	110
-0	100	111
0	000	000
1	001	001
2	010	010
3	011	011



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Problem 6 (15 points)
Compute $z = (a - b) + (c - d)$ in 2's complement.

(a) (4 points) Fill up the table given below.

(b) (2 points) How many bits should z have to represent the correct result?

(c) (9 points) Perform calculations on bit-vectors representing a, b, c and d (all in 2's complement) and show every step of your work. z should be given in **both** decimal representation and 2's complement. **If you want**, you can extend the table below.

(a)

	Decimal	Sign-Magnitude	2's Complement
a	-13	1 1101	10011
b	-10	1 1010	10110
c	-6	1 110	1010
d	82	0 1010010	01010010

(b) 8 bits are needed

(c) $z = (a-b) + (c-d)$ $-b = 01010$

$= a + -b + c + -d$ $-d = 10101110$

$a + -b =$ 10011
+ 01010
11101

$c + -d =$ 11111010
+ 10101110
110101000

$a-b$ 11111101
$+ c-d$ + 10101000
110100101

10100101

Decimal:

$-13 - (-10) = -3$

$-6 - 82 = -88$

$-3 + -88 = -91$

-91 91 − 64 = 27, 27 − 16 = 11, 11 − 8 = 3

sign
$= 01011011$

-91

$= 10100100 + 1$

$= 10100101$

0

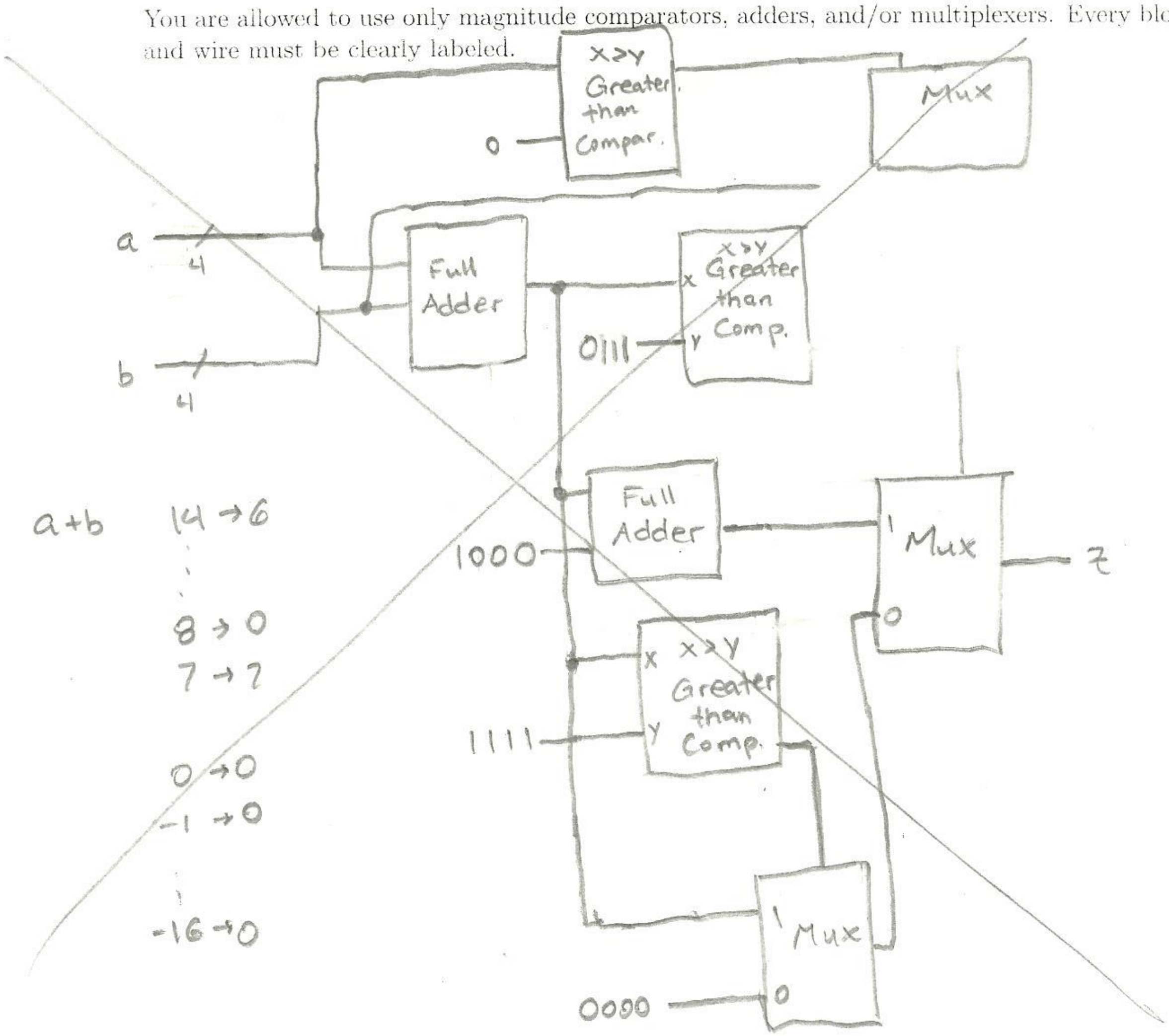
Problem 7 (15 points)

Design a combinational network that has 4-bit inputs a and b, and a four-bit output z. All inputs and outputs are given in two's complement representation. The function of the system is

$$z = \max\{a + b, 0\} \mod 8$$

For example, if $a = 3$ and $b = 7$, then $z = 2$. If $a = 3$ and $b = -7$, then $z = 0$.

You are allowed to use only magnitude comparators, adders, and/or multiplexers. Every block and wire must be clearly labeled.



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