check 2 & 3

Problem 1 (10 points)
Reduce the following expression using Boolean algebra postulates and theorems. Show intermediate steps and mention the rules used at each step.

$$f(w,x,y,z) = \overline{\bar{x}\bar{y} + \bar{z}} + z + xy + wz$$

$\overline{\bar{x}\bar{y} + \bar{z}} + z + xy + wz$

$(x+y)\bar{z} + z + xy + wz$ — DeMorgan's

$x\bar{z} + y\bar{z} + z + xy + wz$ — distributive

$z + wz + xy + x\bar{z} + y\bar{z}$ — communative

$z + xy + x\bar{z} + y\bar{z}$ — absorption

$z + xyz + xy\bar{z} + x\bar{z} + y\bar{z}$ — combining (reverse)

$z(1+xy) + \bar{z}(xy + x + y)$ — distributive

$z(1) + \bar{z}(x+y)$ — absorption $(1+a=1)$

$\overline{\overline{z + \bar{z}(x+y)}}$ — double complement (identical)

$\overline{(\bar{z})(z + \bar{x}\bar{y})}$ — DeMorgan's

$\overline{(\bar{z}z + \bar{z}\bar{x}\bar{y})}$ — distributive

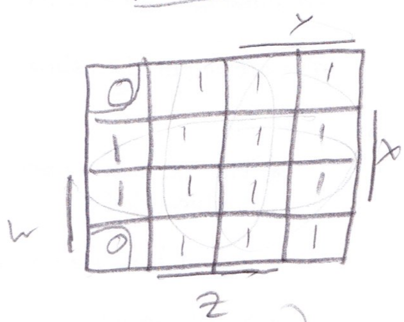
$\overline{(0 + \bar{z}\bar{x}\bar{y})}$ — identity $(aa'=0)$

$\overline{(\bar{z}\bar{x}\bar{y})}$ — identity $(0+a=a)$

$z + x + y$ — De morgans

$\boxed{x + y + z}$ — communative

to check



$(x+y+z)$ $x + z + y$

✓ 10

15

Problem 2 (15 points)
Consider the following function

$$f(a, b, c, d) = \Sigma m(1, 3, 4, 9, 11, 13).$$

(a) (8 points) Use K-maps to minimize **both** the sum of products and products of sums form. Write the simplified Boolean expressions.

(b) (7 points) Implement the function using minimal number of gates. You can use either NOR gates or NAND gates with maximum number of four inputs. Assume that inputs are available in both uncomplemented and complemented form.

c, b, a, d

0	1	1	0
1	0	0	0
0	1	0	0
0	1	1	0

SoP

$b'd + ac'd + a'bc'd'$

sum of products

3 ands, 1 or → 4 nands

c, b, a, d

0	1	1	0
1	0	0	0
0	1	0	0
0	1	1	0

PoS

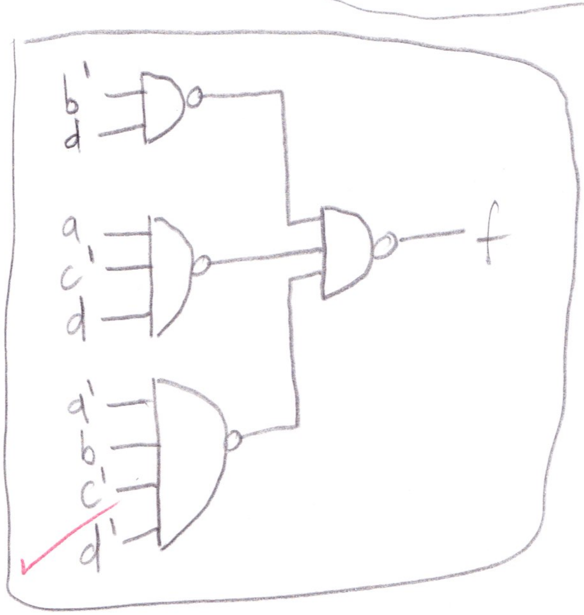
$(c'+d)$ is not essential & not needed

$(b+d)(a'+d)(b'+c')(a+b'+d')$

product of sums

4 or's, 1 and → 5 nors

<



$\overline{\overline{b'd + ac'd + a'bc'd'}}$

↓

$\overline{(\overline{b'd})(\overline{ac'd})(\overline{a'bc'd'})}$ → nand-nand.

8

Problem 3 (20 points)

Consider the following function

$$f(a,b,c,d) = \overline{(\bar{a}b + d)(\bar{b} + \bar{d})}.$$

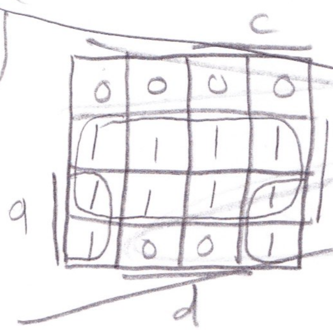
(a) (6 points) Find minimal sum-of-products and products-of-sums expressions for the switching expression f.

(b) (8 points) Determine the prime implicants and the essential prime implicants for this expression.

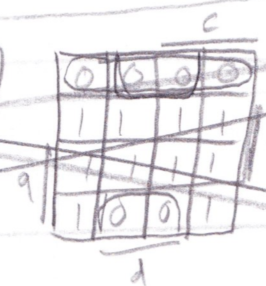
(c) (6 points) Does this function have a unique minimal sum-of-products? If not, list any other minimal sum-of-products expressions. Does this function have a unique minimal product-of-sums? If not, list any other minimal product-of-sums expressions.

$\overline{(\bar{a}b+d)(\bar{b}+\bar{d})} = (\overline{\bar{a}b+d}) + (\overline{\bar{b}+\bar{d}}) = ((a+\bar{b})\cdot\bar{d}) + (bd) = a\bar{d} + \bar{b}\bar{d} + bd$

a)



~~Sum of products~~ ~~$b + ad'$~~



~~Product of Sums~~ ~~$(a+b)(b+d')$~~

b) ~~Prime implicants: b, ad'~~
~~Essential prime implicants: b, ad'~~

~~both are prime & contain minterms not included in any other implicant~~

c) This function has a unique minimal sum-of-products & product-of-sums.

Due to all the prime implicants & implicates being essential.

correct a & b are on other page

a)

			c	c	
	1	0	0	1	
	0	1	1	0	b
a	1	1	1	1	b
a	1	0	0	1	
		d	d		

→ $\boxed{\underline{\text{sum of products}} \quad bd + \bar{b}\bar{d} + a\bar{d}}$ ✓

			c	c	
	1	0	0	1	
	0	1	1	0	b
a	1	1	1	1	b
a	1	0	0	1	
		d	d		

→ $\boxed{\underline{\text{product of sums}} \quad (b+d')\cdot(a+b'+d)}$ ✓

b) prime implicants: bd, $\bar{b}\bar{d}$, $a\bar{d}$, ab ← all are prime & essential

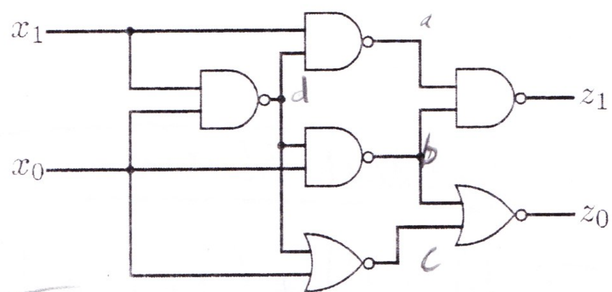
EPI: bd, $\bar{b}\bar{d}$, $a\bar{d}$ ✗

c) see first page

Problem 4 (10 points)

10

Analyze the NAND-NOR network shown in the figure below. Obtain switching expressions for the outputs.



$$z_1 = \overline{ab} = \overline{(\overline{x_1 d})(\overline{x_0 d})} = x_1 d + x_0 d = d(x_1 + x_0) = \overline{x_0 x_1}(x_1 + x_0)$$

$$= (\bar{x}_0 + \bar{x}_1)(x_0 + x_1)$$

$$= (\bar{x}_0 + \bar{x}_1)x_0 + (\bar{x}_0 + \bar{x}_1)x_1$$

$$= \bar{x}_0 x_0 + \bar{x}_1 x_0 + \bar{x}_0 x_1 + \bar{x}_1 x_1$$

$$= 0 + x_0\bar{x}_1 + \bar{x}_0 x_1 + 0$$

$$= x_0\bar{x}_1 + \bar{x}_0 x_1$$

$$= x_0 \oplus x_1 = z_1$$

$$z_0 = \overline{b + c}$$

$$a = \overline{x_1 d} = \overline{x_1(\overline{x_0 x_1})} = \bar{x}_1 + (x_0 x_1)$$

$$b = \overline{x_0 d} = \overline{x_0(\overline{x_0 x_1})} = \bar{x}_0 + (x_0 x_1)$$

$$c = \overline{x_0 + d} = \overline{x_0 + (\overline{x_0 x_1})} = \bar{x}_0(x_0 x_1)$$

$$d = \overline{x_0 x_1}$$

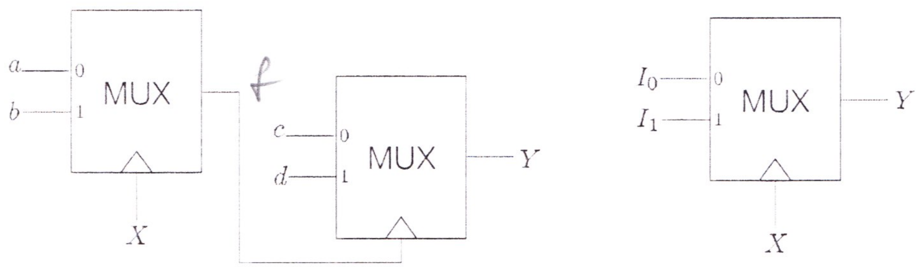
$$z_0 = \overline{\overline{x_0 d} + \overline{x_0 + d}} = (x_0 d)(x_0 + d) = x_0 x_0 d + x_0 d d = x_0 d + x_0 d = x_0 d$$

$$x_0(\overline{x_0 x_1}) = x_0(\bar{x}_0 + \bar{x}_1) = x_0\bar{x}_0 + x_0\bar{x}_1 = 0 + x_0\bar{x}_1$$

$$= x_0\bar{x}_1$$

$$\boxed{\begin{aligned} z_1 &= x_0\bar{x}_1 + \bar{x}_0 x_1 \\ z_0 &= x_0\bar{x}_1 \end{aligned}} \quad z_1 = x_0 \oplus x_1$$

Problem 5 (15 points)
Consider the two circuits



(a) (8 points) Find I_0, I_1, so that both circuits are equivalent.

(b) (7 points) Write the Boolean expression of Y in terms of X, a, b, c, and d.

a) left side

$Y = c\bar{f} + df$

$f = a\bar{X} + bX$

$Y = c(a\bar{x} + bx) + d(a\bar{x} + bX)$

$= ca\bar{x} + cbx + da\bar{x} + dbx$

$= (ca + da)\bar{x} + (cb + db)x$

$I_0\bar{x} + I_1 x = (ca + da)\bar{x} + (cd + db)x$

right side

$Y = I_0\bar{X} + I_1 X$

$\boxed{\begin{aligned} I_0 &= ca + da \\ I_1 &= cd + db \end{aligned}}$ $\begin{aligned} &= a(c+d) \\ &= d(c+b) \end{aligned}$

b) $\boxed{Y = ca\bar{x} + cbx + da\bar{x} + dbx}$

(from part A)

a) 4

b) 3

15

Problem 6 (15 points)

Design a combinational network that has a three-bit input x representing the digits 0 to 7, and a three-bit output y representing the same set of integers. The function of the system is

$$y = (3x + 1) \mod 8.$$

Use a binary decoder, a binary encoder, ~~and (one or more) OR gates~~.

X	Y
0	1
1	4
2	7
3	2
4	5
5	0
6	3
7	6

$(0+1) \mod 8 = 1$

$(3+1) \mod 8 = 4$

etc.

$(6+1) \mod 8 \rightarrow 7$

$(9+1) \mod 8 \rightarrow 10 \mod 8 = 2$

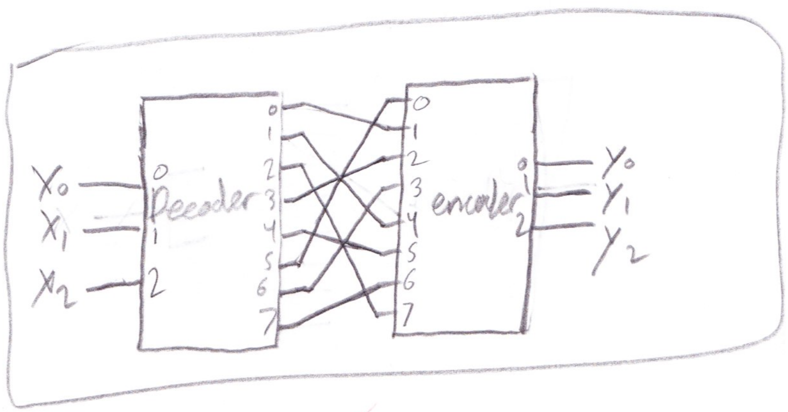
$(12+1) \mod 8 \rightarrow 13 \mod 8 = 5$

$(15+1) \mod 8 \rightarrow 16 \mod 8 = 0$

$(18+1) \mod 8 \rightarrow 19 \mod 8 = 3$

$(21+1) \mod 8 \rightarrow 22 \mod 8 = 6$

Simply remap to this using the decoder & encoder



Problem 7 (15 points)
Compute $z = a + 2b - c$ in 2's complement for $a = -7$, $b = 12$, and $c = -97$. Perform calculations on bit-vectors representing a, b, and c and show every step of your work. How many bits should z have to represent the correct result? Check your work by showing, for each step, the corresponding values in decimal number system.

$z = a + 2b - c$

$z = -7 + 2(12) - (-97)$

$z = -7 + 24 + 97$

$z = 24 + 90$

$z = 114$ ← in 2's compliment, will need $\boxed{8 \text{ bits}}$ enough to represent 114

8 bits → 127 to −128

$a = -7 = \text{neg}(0000\ 0111) = 1111\ 1000 + 1 = 1111\ 1001 = a = 256 - 7 = 249$

$b = 12 = 0000\ 1100 = b$

$c = -97 = \text{neg}(0110\ 0001) = 1001\ 1110 + 1 = 1001\ 1111 = c = 256 - 97$

$97 = 64 + 32 + 1$

$z = a + 2b - c$

$z = \underbrace{a + b \text{ (bit shifted left by 1)}} + (-c)$

$$\begin{array}{r} 1111\ 1001 \\ +\ 0001\ 1000 \\ \hline \cancel{1}0001\ 0001 \end{array} = \begin{array}{r} -7 \\ +24 \\ \hline 17 \end{array}$$

ignore

$10011111 \xrightarrow{\text{invert bits \& add 1}} 01100000 + 1$
$01100001 = 97$

$z = 00010001 + (-c)$

$z = 17 + (-c)$

$z = 17 + (-(-97))$

$z = 17 + 97$

$$z = \begin{array}{r} 00010001 \\ +\ 01100001 \\ \hline 01110010 \end{array} = \begin{array}{r} 17 \\ +97 \\ \hline 114 \end{array}$$

→ correct

$\boxed{01110010} = 64 + 32 + 16 + 2$
$= 80 + 34$
$= 114$

$\boxed{8 \text{ bits}}$

15