EEM16: Logic Design of Digital Systems

Fall 2015 Midterm
Wednesday, October 28, 2015
Time Limit: 110 Minutes

Name (Print):
ID Number:

This exam contains 15 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You are required to show your work on each problem on this exam. The following rules apply:

- The exam is closed book. You are allowed one double-sided $8\frac{1}{2} \times 11''$ double-sided cheat sheet.
- Calculators are not allowed.
- Show the intermediate steps leading to your final solution for each problem. There will be NO partial credit for work done correctly using a wrong answer from a previous part of a question. For example, if part a) is wrong and part b) depends on part a), then part b) will be wrong. Therefore, be very careful and double check your work!
- You can use both sides of the sheets to answer questions.

Do not write in the table to the right.

Problem	Points	Score
1	10	10
2	15	11
3	20	20
4	10	10
5	15	15
6	15	15
7	15	15
Total:	100	96

Problem 1 (10 points)

Reduce the following expression using Boolean algebra postulates and theorems. Show intermediate steps and mention the rules used at each step.

$$f(w,x,y,z) = \overline{\bar{x}\bar{y} + z} + z + xy + wz$$

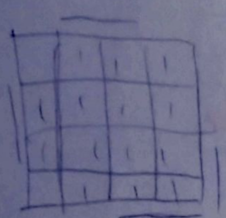
$= (x+y)\bar{z} + z + xy + wz$ (DeMorgan's law)

$= (x+y)\bar{z} + z + xy$ (combining)

$= \overline{\overline{(x+y)\bar{z} + z}} + xy$ $(\bar{\bar{a}} = a)$

$= \overline{\overline{(x+y)\bar{z}}\,\bar{z}} + xy$ (DeMorgan's law)

$= \overline{(\overline{(x+y)} + z)\bar{z}} + xy$ (DeMorgan's law)



x+y+z

$= \overline{\overline{(x+y)}\bar{z} + z\bar{z}} + xy$ (distributive property)

$= \overline{\overline{(x+y)}\,\bar{z}} + xy$ $(a\bar{a} = 0)$

✓ $= x+y+z + xy$ (DeMorgan's law)

$= \boxed{x+y+z}$ (combining)

10

Problem 2 (15 points)
Consider the following function

$$f(a, b, c, d) = \Sigma m(1, 3, 4, 9, 11, 13).$$

4 (a) (8 points) Use K-maps to minimize **both** the sum of products and products of sums form. Write the simplified Boolean expressions.

7 (b) (7 points) Implement the function using minimal number of gates. You can use either NOR gates or NAND gates with maximum number of four inputs. Assume that inputs are available in both uncomplemented and complemented form.

a)

a

0	1	1	0
1	0	0	0
0	1	0	0
0	1	1	0

c

d

b

Labels!!

Sum of products:

$$f = \bar{a}\bar{b}c\bar{d} + a\bar{b}d + a\bar{c}$$??

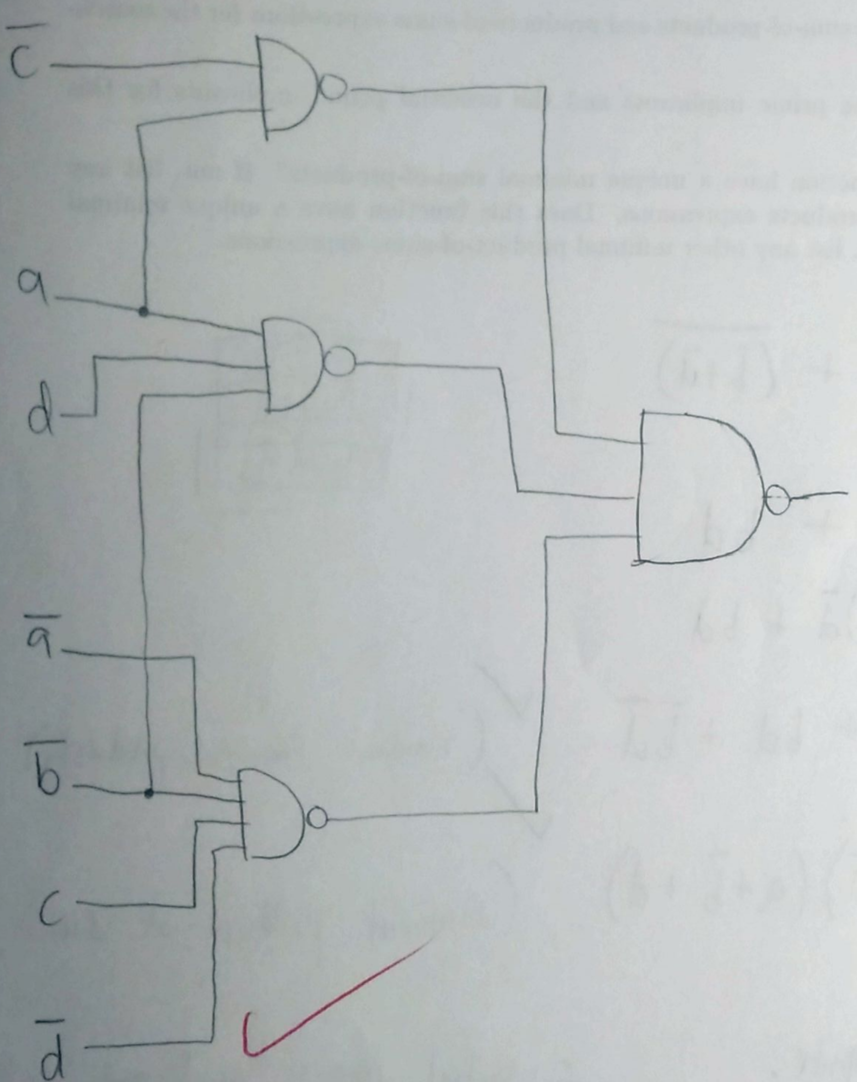
This is correct if the labels are as shown in the k-map.

product of sums:

$$f = (a+b+\bar{d})(\bar{a}+\bar{c}+d)(a+c)(\bar{b}+\bar{c})$$

b) $$f = \overline{\overline{\bar{a}\bar{b}c\bar{d} + a\bar{b}d + a\bar{c}}}$$

$$= \overline{\overline{(\bar{a}\bar{b}c\bar{d})}\;\overline{(a\bar{b}d)}\;\overline{(a\bar{c})}}$$


$\overline{c}$
a
d
f
$\overline{a}$
$\overline{b}$
c
$\overline{d}$

Problem 3 (20 points)

Consider the following function

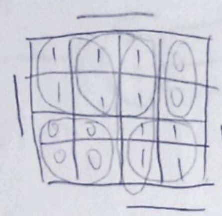
$$f(a,b,c,d) = \overline{(\bar{a}b + d)(\bar{b} + \bar{d})}.$$

(a) (6 points) Find minimal sum-of-products and products-of-sums expressions for the switching expression f.

(b) (8 points) Determine the prime implicants and the essential prime implicants for this expression.

(c) (6 points) Does this function have a unique minimal sum-of-products? If not, list any other minimal sum-of-products expressions. Does this function have a unique minimal product-of-sums? If not, list any other minimal product-of-sums expressions.

a) $f = \overline{(\bar{a}b+d)} + \overline{(\bar{b}+\bar{d})}$



$= \overline{\bar{a}b}\,\bar{d} + bd$

$= (a+\bar{b})\bar{d} + bd$

$= a\bar{d} + bd + \bar{b}\bar{d}$ (minimal sum-of-products)

$= (b+\bar{d})(a+\bar{b}+d)$ (minimal product-of-sums)

b) prime implicants:

$\bar{b}\bar{d}$, $a\bar{d}$, ab, bd

essential prime implicants:

$\bar{b}\bar{d}$, bd

(minterms 0, 4) (minterms 10, 14)

c) no unique minimal sum-of-products:

$f = ab + bd + \bar{b}\bar{d}$

f has a unique minimal product-of-sums.

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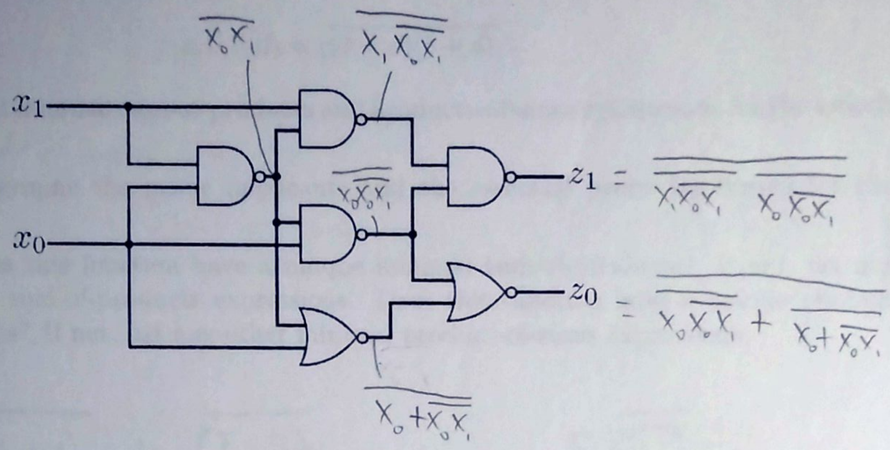
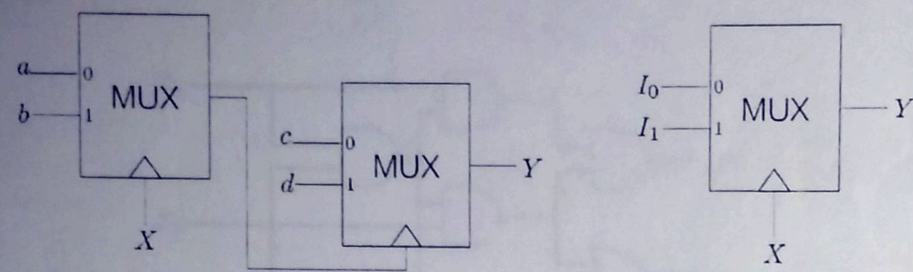
Problem 4 (10 points)
Analyze the NAND-NOR network shown in the figure below. Obtain switching expressions for the outputs.



$$Z_0 = \left(x_0 \overline{x_0 x_1}\right)\left(x_0 + \overline{x_0 x_1}\right)$$
$$= x_0 \overline{x_0 x_1} + x_0 \overline{x_0 x_1}$$
$$= x_0 \overline{x_0 x_1}$$
$$= x_0 \left(\overline{x_0} + \overline{x_1}\right)$$
$$\boxed{Z_0 = x_0 \overline{x_1}}$$

$$Z_1 = x_1 \overline{x_0 x_1} + x_0 \overline{x_0 x_1}$$
$$= x_1(\overline{x_0} + \overline{x_1}) + x_0(\overline{x_0} + \overline{x_1})$$
$$\boxed{Z_1 = \overline{x_0} x_1 + x_0 \overline{x_1}}$$
$$(= x_0 \text{ XOR } x_1)$$

Problem 5 (15 points)
Consider the two circuits



(a) (8 points) Find I_0, I_1, so that both circuits are equivalent.

(b) (7 points) Write the Boolean expression of Y in terms of X, a, b, c, and d.

a)

X	a	b	Y
0	0	X	c
0	1	X	d
1	X	0	c
1	X	1	d

$\Rightarrow$

X	Y
0	$\bar{a}c + ad$
1	$\bar{b}c + bd$

So $\boxed{I_0 = \bar{a}c + ad \quad I_1 = \bar{b}c + bd}$ ✓ 8

b) $\boxed{Y = \bar{X}(\bar{a}c + ad) + X(\bar{b}c + bd)}$

$(\bar{X} I_0 + X I_1)$

✓ 7

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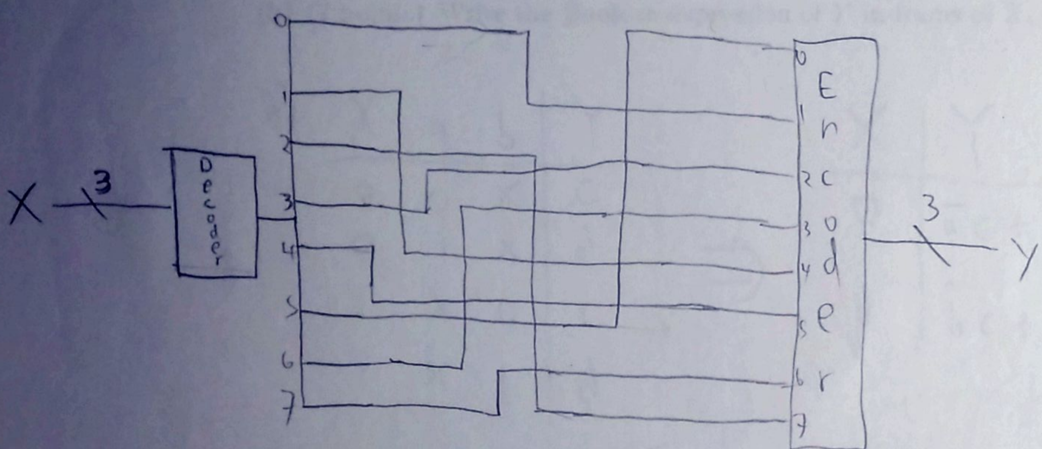
Problem 6 (15 points)

Design a combinational network that has a three-bit input x representing the digits 0 to 7, and a three-bit output y representing the same set of integers. The function of the system is

$$y = (3x + 1) \mod 8.$$

Use a binary decoder, a binary encoder, and or ~~(one or more)~~ OR gates.



X	Y
0	1
1	4
2	7
3	2
4	5
5	0
6	3
7	6

Problem 7 (15 points)
Compute $z = a + 2b - c$ in 2's complement for $a = -7$, $b = 12$, and $c = -97$. Perform calculations on bit-vectors representing a, b, and c and show every step of your work. How many bits should z have to represent the correct result? Check your work by showing, for each step, the corresponding values in decimal number system.

decimal

$12 = b = 01100_{2's\,c}$

$24 = 2b = b \cdot 10_2 = 011000_{2's\,c}$

$7 = -a = 0111_{2's\,c}$

$-7 = a = 1001_{2's\,c}$

$97 = -c = 97 = 01100001_{2's\,c}$

$-97 = c = 10011111_{2's\,c}$

$a + 2b - c = a + 2b + (-c)$

-7	11111001	(sign-extend a)
24	00011000	(sign-extend b)
97	+ 01100001	(make sure c has right sign)
114 =	$\boxed{z = 01110010_{2's\ complement}}$	

Theoretically, if a,b,c were unknown 8-bit quantities, z would need 10 bits.

Here, $a+2b-c = +114 \leq 127$, so $\boxed{\text{8 bits suffice to represent z correctly}}$

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