ECE M16/CS M51A Midterm

Yangchao Wu

TOTAL POINTS

96.5 / 105

QUESTION 1

1 Question 1-a 5 / 5

✓ **- 0 pts** **Correct**

- 1 pts Missing 2x2 PI top/bottom right corner

- 1 pts Missing bottom row PI

- 1 pts Missing Upper Center PI

- 1 pts Missing Lower Center PI

- 1 pts Missing Right Center PI

- 1 pts Introducing additional wrong implicants (not PIs)

- 5 pts Ambiguous circling.

QUESTION 2

2 Question 1-b 4 / 5

- 0 pts Correct

- 1 pts Missed center EPI

- 1 pts Missed right center EPI

- 1 pts Missed 2x2 top/bottom-right corner EPI

✓ **- 1 pts** **Missed bottom row EPI**

- 0.5 pts Missed bottom row EPI but consistent with previous answer

- 1 pts Mistake made with EPI expression (circle is translated wrong)

- 1 pts Inconsistent with previous answer (given previous answer the answer given here is wrong)

- 0.5 pts Missed 2x2 top/bottom right EPI but consistent with previous answer

- 0.5 pts * + notation error

- 5 pts No answer

- 2 pts A should be B, C should be D

QUESTION 3

3 Question 1-c 5 / 5

✓ **- 0 pts** **Correct**

- 1 pts Missed 2x2 bottom right corner

- 0.5 pts Expression is wrong from the circle. (Wrong translation from circle).

- 1 pts Expression inverses are wrong (dual form is given).

- 1 pts Unnecessary PI included

- 2 pts POS is given when SOP is asked.

- 1 pts Top row PI is missed.

- 1 pts Left center PI is missed

- 1 pts Center PI is missed.

- 1 pts Inverse at the beginning is wrong.

- 5 pts Should have started with 0s.

- 2 pts A should be B and C should be D

- 0.5 pts Click here to replace this description.

- 1 pts Simplifications are wrong.

- 4 pts Formed correct POS but nothing else.

QUESTION 4

4 Question 2-a 5 / 5

+ 1 pts Answered yes.

+ 1 pts Wrote the correct form

+ 3 pts Provided proof

- 1 pts Proof incomplete

✓ **+ 5 pts** **All correct**

QUESTION 5

5 Question 2-b 6 / 6

+ 1 pts Wrote XOR to Boolean

+ 2 pts Expanded the Boolean expression

+ 1 pts Simplified the expanded expression

✓ **+ 6 pts** **Got the final DNF**

+ 0 pts Nothing

- 1 pts Did not simplify / not DNF / at least one term is wrong

QUESTION 6

6 Question 2-c 6 / 6

- ✓ **+ 6 pts Got the minimal terms**
- **+ 2 pts** Built on the previous answer
- **+ 1 pts** Drew the k-map
- **+ 2 pts** Verified using k-map
- **+ 0 pts** Nothing
- **- 1 pts** minor mistake

QUESTION 7

7 Question 2-d 7 / 8

- **+ 2 pts** Used NOR for other logics
- **+ 2 pts** Gate diagram for previous logic
- ✓ **+ 8 pts All correct**
- **+ 0 pts** Nothing
- ✓ **- 1 pts Not minimum / minor issue**

① wiring of C seems problematic

QUESTION 8

8 Question 3-a 0 / 4

- **+ 4 pts** Correct
- ✓ **+ 0 pts Wrong**

QUESTION 9

9 Question 3-b 3 / 3

- ✓ **+ 3 pts correct**
- **+ 0 pts** wrong

QUESTION 10

10 Question 3-c 4 / 4

- ✓ **+ 4 pts correct**
- **+ 0 pts** wrong
- **+ 2 pts** partially correct from previous wrong answer
- **+ 1 pts** taking 4's complement

QUESTION 11

11 Question 3-d 3 / 3

- ✓ **+ 3 pts Correct**
- **+ 0 pts** Wrong

QUESTION 12

Question 3-e 11 pts

12.1 3-e-1 4 / 4

- ✓ **+ 4 pts Correct**
- **+ 0 pts** Wrong

12.2 3-e-2 4 / 4

- ✓ **+ 4 pts Correct**
- **+ 0 pts** Wrong
- **+ 2 pts** partially correct from previously wrong answer
- **+ 1 pts** Taking 19's complement

12.3 3-e-3 3 / 3

- ✓ **+ 3 pts Correct**
- **+ 0 pts** Wrong
- **+ 1 pts** partially correct

QUESTION 13

13 Question 4-a 8 / 8

- ✓ **+ 8 pts Correct**
- **+ 4 pts** Used blocks/non-gate components but correct output
- **+ 4 pts** Carry wrong
- **+ 4 pts** Difference output wrong
- **+ 0 pts** No design
- **+ 0 pts** Wrong output for carry and difference

QUESTION 14

14 Question 4-b 6 / 6

- ✓ **+ 6 pts Correct**
- **+ 3 pts** Didn't use half-subtractors but correct output
- **+ 3 pts** carry out wrong
- **+ 3 pts** difference wrong
- **+ 0 pts** Wrong/No design

QUESTION 15

15 Question 4-c 6 / 7

- **+ 7 pts** Correct
- ✓ **+ 6.5 pts Missing first cin but otherwise correct**
- ✓ **- 0.5 pts Missing labels for the bit order (order does matter)**
- **+ 5 pts** Close design but not correct
- **+ 5 pts** Subtracted cin from all bits
- **+ 3 pts** Incomplete/Significant design issues but on

right track

+ **2 pts** Implemented wrong device
- **0.5 pts** Implemented using half-sub instead of full-sub
+ **0 pts** Wrong/No design

QUESTION 16

16 Question 4-d 13 / 14

✓ + **2 pts** **Correct addition function**
✓ + **2 pts** **Correct subtraction function**
✓ + **3 pts** **Correct negation function**
✓ + **2 pts** **Correct multiplication function**
+ **3 pts** MUXs used properly for switching outputs
✓ + **2 pts** **Completed design/put together properly/correct outputs**
- **0.5 pts** Unclear which wires go where/splitting of wires
- **0.5 pts** Unclear which inputs are which for functions
- **0.5 pts** Wrong codes correspond to functions
- **0.5 pts** Wrong names for adder/subtractor
+ **0 pts** No Design

+ **2 Point adjustment**

carry bit not handled properly +2 partial credit

② need a mux (not the max of all carrys)

QUESTION 17

17 Question 5 4.5 / 5

+ **5 pts** Correct
✓ + **4.5 pts** **Correct design idea; minor mistake**
+ **4 pts** Correct design idea; wrong codes match
+ **3 pts** Correct design idea; not complete/significant mistake
+ **3 pts** Gray-Binary converters wrong
+ **2 pts** Did not handle Gray codes properly/Wrong idea for codes
+ **1 pts** Gray-binary converter only/Fundamentally wrong design
+ **0 pts** No/Wrong design

③ Need to specify a 4-bit FA and the third input...

Midterm Exam

Name (Last, First): Wu, Yangchao

Student Id #: 905143259

Student to Left: Zhao, Liqi

Student to Right: Chandrasekar, Kaeshav

Do not start working until instructed to do so.

1. *You must answer in the **space provided** for answers after every question. We will ignore answers written anywhere else in the booklet. **All pages in this booklet must be accounted** for otherwise it will not be graded.*
2. *You are permitted 1 page of notes 8.5x11 (front and back).*
3. *You may not use any electronic device.*

Following table to be filled by course staff only

	Maximum Score	Your Score
Question 1	15	
Question 2	25	
Question 3	25	
Question 4	35	
Question 5 (EC)	+5	
TOTAL	100	

Question #1 (15 pts)

Consider the following Karnaugh Map for the Boolean function, Y. A blank truth table is provided for your convenience.

AB

CD

	"00"	"01"	"11"	"10"
"00"	0	0	✓1	1
"01"	0	1 ✓	0	1 ✓
"11"	0	1	0	0
"10"	1	1	X	X

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	X
1	1	1	1	0

(a) Circle the prime implicants on the map. (5 pts)
How many prime implicants are there? 5

(b) Write the Boolean (sum-of-product) expression of the *essential* prime implicants of (b) (if any). (5 pts)

EssentialPrimeImplicants = $a\bar{d} + ab\bar{c} + \bar{a}bd$

(c) Express as a minimal sum of product, $\neg Y$. (5 pts)
The K-map is provided for your convenience.

AB

CD

	"00"	"01"	"11"	"10"
"00"	0	0	1	1
"01"	0	1	0	1
"11"	0	1	0	0
"10"	1	1	X	X

$\neg Y =$ $ca + abd + \bar{c}\bar{d}\bar{a} + \bar{a}\bar{b}d$

Question #2 (25 pts)

(a) Is DeMorgan's theorem still true with more than two variables? If so, prove it in the case of three variables x, y and z. (5 pts) prove by equality of truth table

Yes.

~~prove x~~

prove $\overline{(xyz)} = \bar{x} + \bar{y} + \bar{z}$

x	y	z	$\overline{(xyz)}$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

x	y	z	$\bar{x}+\bar{y}+\bar{z}$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	
1	1	1	0

(b) Rewrite the following Boolean equation in (Disjunctive) Normal form. (6 pts)

$$f = \overline{A \oplus B} + \overline{B \oplus C}$$

where ⊕ means XOR operation, i.e., $A \oplus B = A\bar{B} + \bar{A}B$

Answer: $f = \overline{A\bar{B} + \bar{A}B} + \overline{B\bar{C} + \bar{B}C}$

$= (\overline{A\bar{B}})(\overline{\bar{A}B}) + (\overline{B\bar{C}})(\overline{\bar{B}C})$

$= (\bar{A} + B)(A + \bar{B}) + (\bar{B} + C)(B + \bar{C})$

$= \cancel{\bar{A}A} + \bar{A}\bar{B} + BA + \cancel{B\bar{B}} + \cancel{\bar{B}B} + \bar{B}\bar{C} + CB + \cancel{C\bar{C}}$ ∵ $\bar{A}A = 0$

$= \bar{A}\bar{B} + BA + \bar{B}\bar{C} + cB = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + BAC + BA\bar{C} + \bar{B}\bar{C}A + \bar{B}\bar{C}\bar{A} + CBA$

$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + BAC + BA\bar{C} + \bar{B}\bar{C}A + CB\bar{A} + CB\bar{A}$

$f =$ ~~$\bar{A}\bar{B} + BA + \bar{B}\bar{C} + CB$~~ $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + BAC + BA\bar{C} + \bar{B}\bar{C}A + CB\bar{A}$

(c) Simplify f from (b) to a minimum sum-of-products. List which Boolean properties you use at each step of the simplification. Hint: you may use K-map to ***verify*** your answer. (6 pts)

Answer: ~~$f = \bar{B}(\bar{A} + \bar{C}) + B(A + C)$~~

$f = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + BAC + BA\bar{C} + \bar{B}\bar{C}A + CB\bar{A}$

c \ ab	00	01	11	10
0	1	0	1	1
1	1	1	1	0

$\bar{a}\bar{b} + \bar{c}a$
$+ cb$

~~$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + BAC + BA\bar{C} + \bar{B}\bar{C}A + \bar{B}\bar{C}\bar{A} + CBA + CB\bar{A}$~~

$ab + c\bar{a}$
$\bar{c}b$

~~$= \bar{A}\bar{B} + BA + \bar{B}\bar{C} + BC$ (combining)~~

$= \bar{A}\bar{B} + BA + \bar{B}\bar{C}A + CB\bar{A}$ (combining)

$= \bar{A}\bar{B} + BA + \bar{B}\bar{C} + \cancel{BC}\, BC$

$= \bar{A}\bar{B} + \bar{C}A + CB$

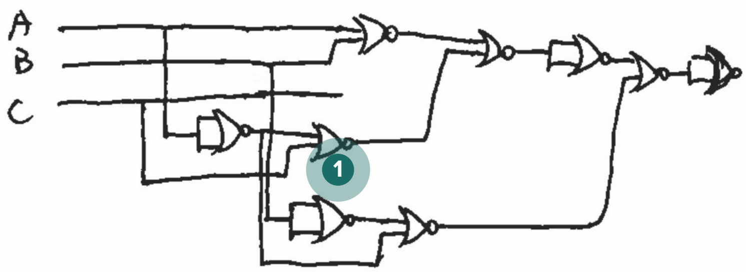
$f =$ ~~$\bar{A}\bar{B} + BA + \bar{B}\bar{C} + BC$~~ ~~$\bar{A}\bar{B} + \bar{C}A + CB$~~

$\bar{A}\bar{B} + \bar{C}A + CB$

(d) With only 2-input NOR gates, implement f with a minimal number of gates. Draw the gate diagram. (Note: no complemented inputs are given) (8 pts)

$$f = \bar{A}\bar{B} + \bar{C}A + CB$$

$$\doteq \overline{(A+B)} + \overline{(C+\bar{A})} + \overline{(\bar{C}+\bar{B})}$$



8+4+1

Question #3 (25 pts)

8+6=14

The following 12-bit word can be used to represent different numbers depending on the encoding

12b'1110_0110_1101

−2048 + 1024 + 512 + 64 + 32 + 8 + 4 + 1 = ~~−1024~~

−1024 −512

(a) If the word is 2's complement, what is the corresponding integer? (4 pts) −275

0001_1001_0011

256+16+2+1

273+3

(b) If we convert the word (treated as unsigned) into base-4, what is the represented number? (3 pts) 321231

(c) If we take answer in (b), extending how we define 1's complement for base-2, write the 3's complement of the base-4 number. (4 pts) ~~0120~~ 012102

(d) What is this word in Hexadecimal? (3 pts) E6D

(e) In base-20 system, assume each digit is now 00, 01, 02, … 09, 10, 11, …19 (each called a "vigit"). For example, 01,19 is 39 in decimal. Using 3 "vigits":

20·20

How would one represent a base-10 integer 1246? (4 pts) 3.2.6

400·3 1200 46

What's the 20's complement representation of -1246 (i.e. the 20's complement of the 1246)? (4 pts) 16. 17. 14

11 18 17 10

16.17.13+1

Using the first vigit as the sign vigit, what is the most positive value in base-10 integer that can be represented? (3 pts) 3999

0 1 2

1 4 5 6

16 17 14

9.19.19

9·400 + 19·20 + 19

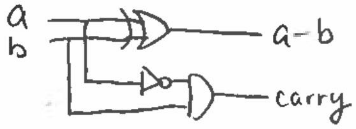
3600 + 380 + 19

3999

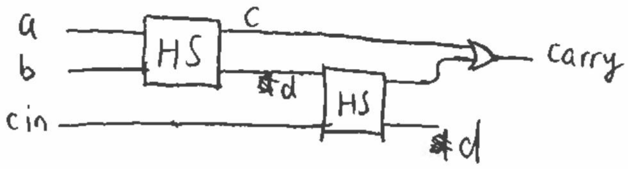
Question #4 (35 pts)

(a) Implement a one-bit "half-subtractor" from gates. The carry-out of this subtractor is 1 when the result is -1. The truth table for this is shown below: (8 pts)

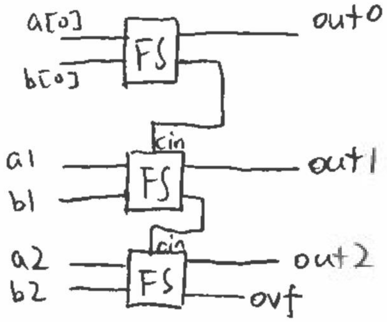
a	b	a - b	"carry"
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0



(b) Implement a "full-subtractor" from "half-subtractor" blocks. (6 pts)



(c) Implement a 3-bit "subtractor" from 1-bit "full-subtractor" blocks. (7 pts)

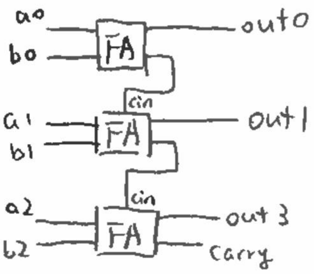


(d) Processors use a block called an ALU (Arithmetic Logic Unit) as part of their processing capability. Here we will implement a very basic ALU with a total of 4 functions, selected by a 2-bit code. Using the building blocks discussed in lecture and the 3-bit subtractor block, implement a 3-bit ALU that can add, subtract, negate one argument, and multiply by 2. The select codes are listed in the table below. Note that there are 3 inputs (3-bit a, 3-bit b, and the 2-bit select code) and 2 outputs (3-bit result and a 1-bit carry). (14 pts)

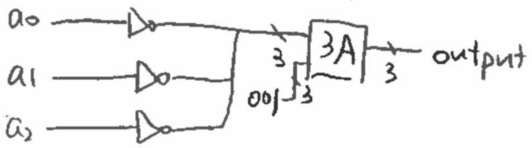
Hint: Multiplying a number is like shifting the bits to the left and using 0 as the lowest bit. An example: a = 1 = 3'b001 -> 2a = 2 = 4'b0010

Select Code	Result (3-bits)	Carry bit
00	a + b	carry out
01	a - b	carry out
10	-a	0
11	2*a	Product MSB

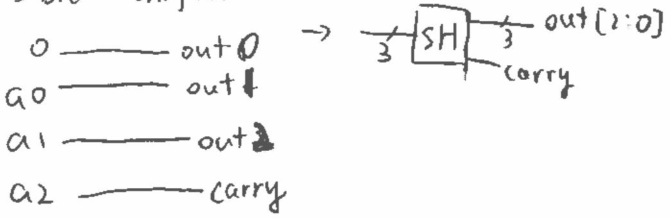
3 bit adder : 3A

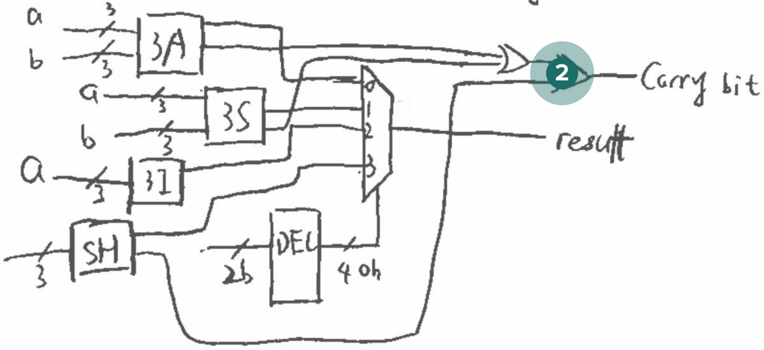


3 bit negator : 3I



3 bit shifter





Question #5 (Extra Credit - 5 pts)

Implement a 4-bit Gray code +1 incrementor using building blocks (no gates). The 4-bit Gray codes are shown below.

Decimal Number	Gray Code
0	0000
1	0001
2	0011
3	0010
4	0110
5	0111
6	0101
7	0100
8	1100
9	1101
10	1111
11	1110
12	1010
13	1011
14	1001
15	1000