ECE M16/CS M51A Midterm

Xiangyu Wan

TOTAL POINTS

91 / 105

QUESTION 1

1 Question 1-a 5 / 5

✓ **- 0 pts Correct**

- **- 1 pts** Missing 2x2 PI top/bottom right corner
- **- 1 pts** Missing bottom row PI
- **- 1 pts** Missing Upper Center PI
- **- 1 pts** Missing Lower Center PI
- **- 1 pts** Missing Right Center PI
- **- 1 pts** Introducing additional wrong implicants (not PIs)
- **- 5 pts** Ambiguous circling.

QUESTION 2

2 Question 1-b 4.5 / 5

- **- 0 pts** Correct
- **- 1 pts** Missed center EPI
- **- 1 pts** Missed right center EPI
- **- 1 pts** Missed 2x2 top/bottom-right corner EPI
- **- 1 pts** Missed bottom row EPI
- **- 0.5 pts** Missed bottom row EPI but consistent with previous answer
- **- 1 pts** Mistake made with EPI expression (circle is translated wrong)
- **- 1 pts** Inconsistent with previous answer (given previous answer the answer given here is wrong)

✓ **- 0.5 pts Missed 2x2 top/bottom right EPI but consistent with previous answer**

- **- 0.5 pts** * + notation error
- **- 5 pts** No answer
- **- 2 pts** A should be B, C should be D

QUESTION 3

3 Question 1-c 5 / 5

✓ **- 0 pts Correct**

- **- 1 pts** Missed 2x2 bottom right corner
- **- 0.5 pts** Expression is wrong from the circle. (Wrong translation from circle).
- **- 1 pts** Expression inverses are wrong (dual form is given).
- **- 1 pts** Unnecessary PI included
- **- 2 pts** POS is given when SOP is asked.
- **- 1 pts** Top row PI is missed.
- **- 1 pts** Left center PI is missed
- **- 1 pts** Center PI is missed.
- **- 1 pts** Inverse at the beginning is wrong.
- **- 5 pts** Should have started with 0s.
- **- 2 pts** A should be B and C should be D
- **- 0.5 pts** Click here to replace this description.
- **- 1 pts** Simplifications are wrong.
- **- 4 pts** Formed correct POS but nothing else.

QUESTION 4

4 Question 2-a 5 / 5

✓ **+ 1 pts Answered yes.**

✓ **+ 1 pts Wrote the correct form**

✓ **+ 3 pts Provided proof**

- **- 1 pts** Proof incomplete
- **+ 5 pts** All correct

QUESTION 5

5 Question 2-b 6 / 6

- **+ 1 pts** Wrote XOR to Boolean
- **+ 2 pts** Expanded the Boolean expression
- **+ 1 pts** Simplified the expanded expression

✓ **+ 6 pts Got the final DNF**

- **+ 0 pts** Nothing
- **- 1 pts** Did not simplify / not DNF / at least one term is wrong

QUESTION 6

6 Question 2-c 2 / 6

+ **6 pts** Got the minimal terms

✓ + **2 pts** **Built on the previous answer**

+ **1 pts** Drew the k-map

+ **2 pts** Verified using k-map

+ **0 pts** Nothing

- **1 pts** minor mistake

QUESTION 7

7 Question 2-d 4 / 8

✓ + **2 pts** **Used NOR for other logics**

✓ + **2 pts** **Gate diagram for previous logic**

+ **8 pts** All correct

+ **0 pts** Nothing

- **1 pts** Not minimum / minor issue

1 gate diagram doesn't match this expression?

QUESTION 8

8 Question 3-a 4 / 4

✓ + **4 pts** **Correct**

+ **0 pts** Wrong

QUESTION 9

9 Question 3-b 3 / 3

✓ + **3 pts** **correct**

+ **0 pts** wrong

QUESTION 10

10 Question 3-c 4 / 4

✓ + **4 pts** **correct**

+ **0 pts** wrong

+ **2 pts** partially correct from previous wrong answer

+ **1 pts** taking 4's complement

QUESTION 11

11 Question 3-d 3 / 3

✓ + **3 pts** **Correct**

+ **0 pts** Wrong

QUESTION 12

Question 3-e 11 pts

12.1 3-e-1 4 / 4

✓ + **4 pts** **Correct**

+ **0 pts** Wrong

12.2 3-e-2 0 / 4

+ **4 pts** Correct

✓ + **0 pts** **Wrong**

+ **2 pts** partially correct from previously wrong answer

+ **1 pts** Taking 19's complement

12.3 3-e-3 3 / 3

✓ + **3 pts** **Correct**

+ **0 pts** Wrong

+ **1 pts** partially correct

QUESTION 13

13 Question 4-a 8 / 8

✓ + **8 pts** **Correct**

+ **4 pts** Used blocks/non-gate components but correct output

+ **4 pts** Carry wrong

+ **4 pts** Difference output wrong

+ **0 pts** No design

+ **0 pts** Wrong output for carry and difference

QUESTION 14

14 Question 4-b 6 / 6

✓ + **6 pts** **Correct**

+ **3 pts** Didn't use half-subtractors but correct output

+ **3 pts** carry out wrong

+ **3 pts** difference wrong

+ **0 pts** Wrong/No design

QUESTION 15

15 Question 4-c 6.5 / 7

✓ + **7 pts** **Correct**

+ **6.5 pts** Missing first cin but otherwise correct

✓ - **0.5 pts** **Missing labels for the bit order (order does matter)**

+ **5 pts** Close design but not correct

+ **5 pts** Subtracted cin from all bits

+ **3 pts** Incomplete/Significant design issues but on right track

+ **2 pts** Implemented wrong device

- **0.5 pts** Implemented using half-sub instead of full-sub

+ **0 pts** Wrong/No design

QUESTION 16

16 Question 4-d 13 / 14

✓ + **2 pts** **Correct addition function**

✓ + **2 pts** **Correct subtraction function**

✓ + **3 pts** **Correct negation function**

✓ + **2 pts** **Correct multiplication function**

✓ + **3 pts** **MUXs used properly for switching outputs**

✓ + **2 pts** **Completed design/put together properly/correct outputs**

✓ - **0.5 pts** **Unclear which wires go where/splitting of wires**

- **0.5 pts** Unclear which inputs are which for functions

- **0.5 pts** Wrong codes correspond to functions

✓ - **0.5 pts** **Wrong names for adder/subtractor**

+ **0 pts** No Design

(2) need to split off wires/show which are which

(3) 3 bit subtractor

QUESTION 17

17 Question 5 5 / 5

✓ + **5 pts** **Correct**

+ **4.5 pts** Correct design idea; minor mistake

+ **4 pts** Correct design idea; wrong codes match

+ **3 pts** Correct design idea; not complete/significant mistake

+ **3 pts** Gray-Binary converters wrong

+ **2 pts** Did not handle Gray codes properly/Wrong idea for codes

+ **1 pts** Gray-binary converter only/Fundamentally wrong design

+ **0 pts** No/Wrong design

Midterm Exam

Name (Last, First): Xiangyu Wan

Student Id #: 805061659

Student to Left: Linfeng Li

Student to Right: Jerry Long

Do not start working until instructed to do so.

1. *You must answer in the* ***space provided*** *for answers after every question. We will ignore answers written anywhere else in the booklet.* ***All pages in this booklet must be accounted*** *for otherwise it will not be graded.*
2. *You are permitted 1 page of notes 8.5x11 (front and back).*
3. *You may not use any electronic device.*

Following table to be filled by course staff only

	Maximum Score	Your Score
Question 1	15	
Question 2	25	
Question 3	25	
Question 4	35	
Question 5 (EC)	+5	
TOTAL	100	

Question #1 (15 pts)

Consider the following Karnaugh Map for the Boolean function, Y. A blank truth table is provided for your convenience.

CD \ AB	"00"	"01"	"11"	"10"
"00"	0	0	1	1
"01"	0	1	0	1
"11"	0	1	0	0
"10"	1	1	X	X

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

(a) Circle the prime implicants on the map. (5 pts)
How many prime implicants are there? 5

(b) Write the Boolean (sum-of-product) expression of the *essential* prime implicants of (b) (if any). (5 pts)

EssentialPrimeImplicants = $(\bar{A}BD)+(C\bar{D})+(A\bar{C}\bar{D})+(A\bar{B}\bar{C})$

(c) Express as a minimal sum of product, $\neg Y$. (5 pts)
The K-map is provided for your convenience.

CD \ AB	"00"	"01"	"11"	"10"
"00"	0	0	1	1
"01"	0	1	0	1
"11"	0	1	0	0
"10"	1	1	X	X

0 X 0 0
0 0 X 1
~~X 0 1 1~~
1 1 X 1
1 X 1 X

$\neg Y = (\bar{A}\bar{C}\bar{D}) + \bar{A}\bar{B}D + AC + ABD$

Question #2 (25 pts)

(a) Is DeMorgan's theorem still true with more than two variables? If so, prove it in the case of three variables x, y and z. (5 pts) yes

$\overline{xyz} = \overline{(xy)z} = \overline{xy} + \bar{z} = \bar{x} + \bar{y} + \bar{z}$

$\overline{(x+y+z)} = \overline{(x+y)+z} = \overline{(x+y)} \cdot \bar{z} = (\bar{x} \cdot \bar{y})\bar{z} = \bar{x}\bar{y}\bar{z}$

using 2 input DeMorgans

(b) Rewrite the following Boolean equation in (Disjunctive) Normal form. (6 pts) Fully

$$f = \overline{A \oplus B} + \overline{B \oplus C}$$

where ⊕ means XOR operation, i.e., $A \oplus B = A\bar{B} + \bar{A}B$

Answer: $\overline{A\bar{B} + \bar{A}B} + \overline{B\bar{C} + \bar{B}C}$

$= (\bar{A}+B)(A+\bar{B}) + (\bar{B}+C)(B+\bar{C})$

$= \bar{A}\bar{B} + AB + \bar{B}\bar{C} + BC$

~~$= B(A+C) + \bar{B}(\bar{A}+\bar{C})$~~

$f =$ ~~AB~~ $\overline{A\bar{B}C + \bar{A}B\bar{C}}$

(c) Simplify f from (b) to a minimum sum-of-products. List which Boolean properties you use at each step of the simplification. Hint: you may use K-map to ***verify*** your answer. (6 pts)

Answer:

$f = \bar{A}\bar{B} + AB + \bar{B}\bar{C} + BC$

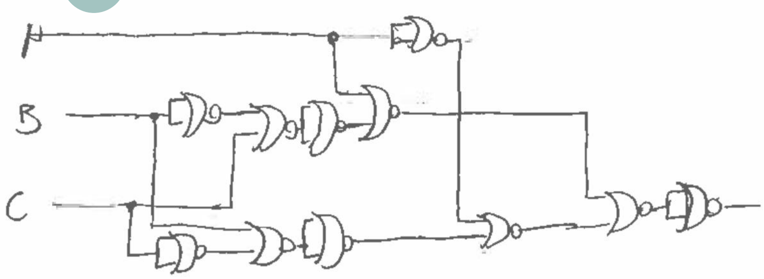
$\overline{B\bar{C}} =$

$B\bar{C} = \overline{\bar{B} + C}$ $\quad \bar{A}\,BC$ $\quad \bar{B}C =$

$\bar{B}C = \overline{B + \bar{C}}$ $\quad \overline{A + \overline{BC}} =$

(d) With only 2-input NOR gates, implement f with a minimal number of gates. Draw the gate diagram. (Note: no complemented inputs are given) (8 pts)

$AB + \bar{A}\bar{B} + BC + \bar{B}\bar{C}$ $\quad \bar{A}B\bar{C} + A\bar{B}C$



8 12+32

44+1=45+64=109-512

Question #3 (25 pts)

The following 12-bit word can be used to represent different numbers depending on the encoding

12b'1110_0110_1101

(a) If the word is 2's complement, what is the corresponding integer? (4 pts) – 403

1+4+8+32+64–512

(b) If we convert the word (treated as unsigned) into base-4, what is the represented number? (3 pts)

$(321231)_4$

(c) If we take answer in (b), extending how we define 1's complement for base-2, write the 3's complement of the base-4 number. (4 pts)

012102

(d) What is this word in Hexadecimal? (3 pts) 0xE6D

(e) In base-20 system, assume each digit is now 00, 01, 02, … 09, 10, 11, …19 (each called a "vigit"). For example, 01,19 is 39 in decimal. Using 3 "vigits":

How would one represent a base-10 integer 1246? (4 pts) 3,2,6

What's the 20's complement representation of -1246 (i.e. the 20's complement of the 1246)? (4 pts) 17.18.15

Using the first vigit as the sign vigit, what is the most positive value in base-10 integer that can be represented? (3 pts) +3999

0, 19, 19

$20^3/2-1 = 4000-1$

20^2-1

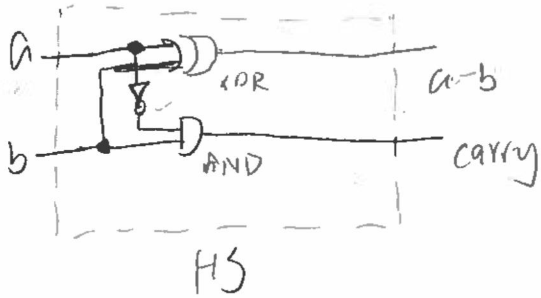
400-1

Question #4 (35 pts)

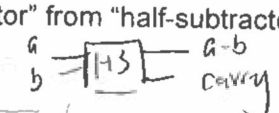
(a) Implement a one-bit "half-subtractor" from gates. The carry-out of this subtractor is 1 when the result is -1. The truth table for this is shown below: (8 pts)

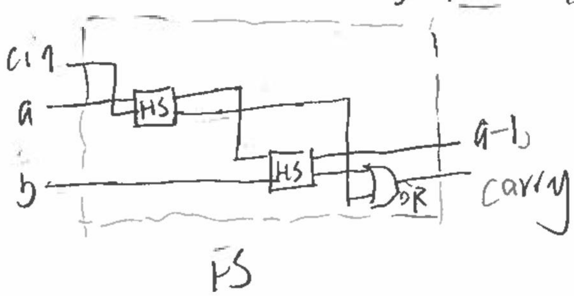
a	b	a - b	"carry"
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

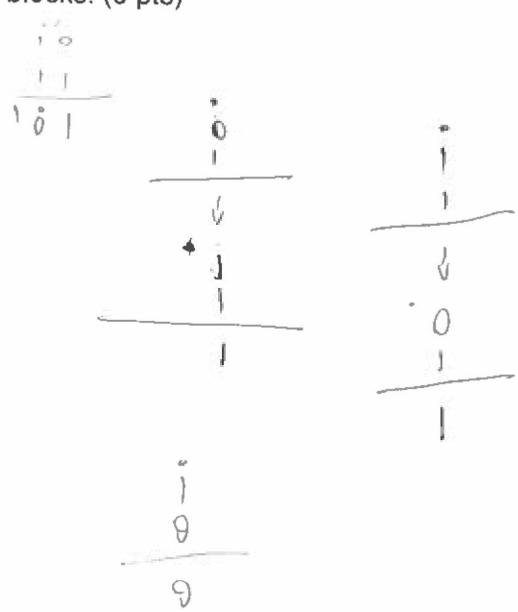
→ $\bar{a}b$



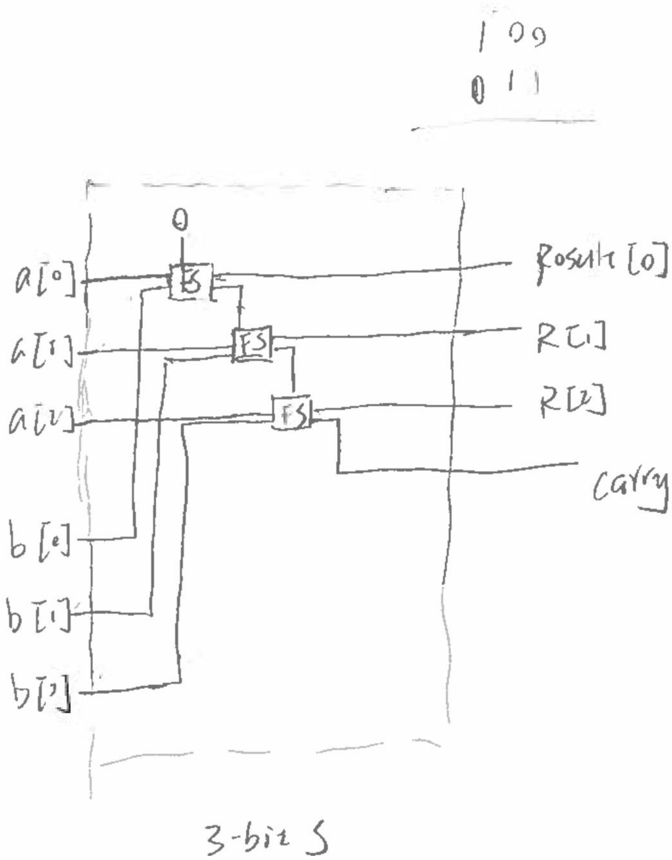
(b) Implement a "full-subtractor" from "half-subtractor" blocks. (6 pts)







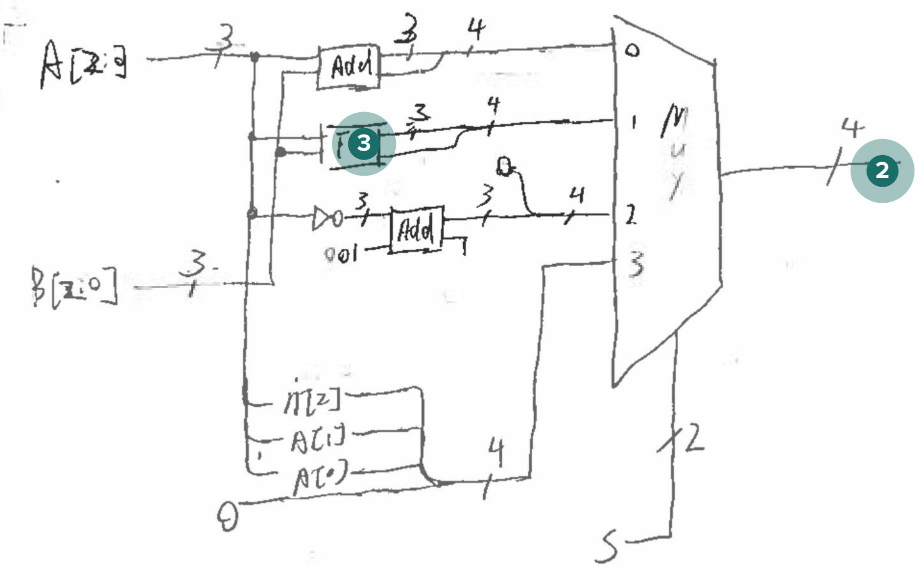
(c) Implement a 3-bit "subtractor" from 1-bit "full-subtractor" blocks. (7 pts)



(d) Processors use a block called an ALU (Arithmetic Logic Unit) as part of their processing capability. Here we will implement a very basic ALU with a total of 4 functions, selected by a 2-bit code. Using the building blocks discussed in lecture and the 3-bit subtractor block, implement a 3-bit ALU that can add, subtract, negate one argument, and multiply by 2. The select codes are listed in the table below. Note that there are 3 inputs (3-bit a, 3-bit b, and the 2-bit select code) and 2 outputs (3-bit result and a 1-bit carry). (14 pts)

Hint: Multiplying a number is like shifting the bits to the left and using 0 as the lowest bit. An example: a = 1 = 3'b001 -> 2a = 2 = 4'b0010

Select Code	Result (3-bits)	Carry bit
00	a + b	carry out
01	a - b	carry out
10	-a	0
11	2*a	Product MSB



Question #5 (Extra Credit - 5 pts)

Implement a 4-bit Gray code +1 incrementor using building blocks (no gates). The 4-bit Gray codes are shown below.

Decimal Number	Gray Code
0	0000
1	0001
2	0011
3	0010
4	0110
5	0111
6	0101
7	0100
8	1100
9	1101
10	1111
11	1110
12	1010
13	1011
14	1001
15	1000