ECE C143A/C243A Midterm

[redacted]

TOTAL POINTS

82.5 / 107

QUESTION 1

Problem 1 30 pts

1.1 (a) 8 / 10

+ **2 pts** i correct
- ✓ + **2 pts** ii correct
- ✓ + **2 pts** iii correct
- ✓ + **2 pts** iv correct
- ✓ + **2 pts** v correct

+ **1 pts** i partial credit
+ **1 pts** ii partial credit
+ **1 pts** iii partial credit
+ **1 pts** iv partial credit
+ **1 pts** v partial credit
+ **0 pts** None correct

1.2 (b) 4 / 4

✓ + **4 pts** **Correct, leads to unidirectional signaling.**

+ **4 pts** Correct, prevents Na+ from flowing inside the cell, enabling the action potential voltage to decrease back to a resting state when K+ ion channels open. Note, it was not enough to say it aided in repolarization, but the particular dynamics of how the inactive state aided needed to be described here to get full points.

+ **2 pts** A true statement about what happens during an action potential, but without an explicit and correct explanation about why the inactivate state contributed to the stated event. Or else some statement related to how repolarization helps it not be as excitable.

+ **2 pts** A statement that it limits the rate of signaling in the nervous system, but without an explicit statement why limiting the firing rate is beneficial.

+ **1 pts** Description of what the inactivate state is (i.e., prevents a channel from staying open indefinitely) but not why it is critical for nervous system signaling. Remember K+ voltage gated ion channels don't have an inactivate state!

+ **1 pts** Incorrect statement that it helps with depolarization (actually, it helps with hyperpolarization), or that it helps with concentration regulation (that's the job of the ion pumps; if you stated that the voltage would not decrease to a level that the voltage gated ion channels would be closed, you received rubric item +2).

1.3 (c) 2.5 / 4

+ **4 pts** Correct, drift and diffusion currents propel the ion into the cell.

✓ + **2.5 pts** **Correct statement that the Na+ ions bond to the binding site because it's more favorable than shedding waters; or statement about related to shedding waters due to the lipid bilayer. These don't receive full credit because the question was asking why doesn't it get stuck to the binding site.**

+ **0 pts** No answer

1.4 (d)(i) 3 / 3

✓ + **3 pts** **Correct, not exponential because it's an inhomogeneous Poisson process**

+ **0 pts** Incorrect.

1.5 (d)(ii) 3 / 3

✓ + **3 pts** **Correct**

+ **2 pts** Statement that the firing rate is constant, but no statement on if this implies exponential ISIs or incorrect statement that the ISIs are not exponential

+ **0 pts** Incorrect

1.6 (d)(iii) 6 / 6

✓ + **6 pts** **Correct, with a justification that the**

variance of the spike counts will be lower since the spike counts are Poisson distributed

+ **4 pts** Correct, but without explicit justification that there are lower variance spike counts

+ **2 pts** Incorrect, with the argument that having more spikes gives a more robust estimate or getting the variance distribution wrong.

+ **2 pts** Incorrect, with an argument for why the neuron is similarly accurate at 10ft and 100ft; the most common one is that the tuning function is injective, but this ignores firing rate variance.

+ **0 pts** Incorrect without justification, or no answer

QUESTION 2

Problem 2 35 pts

2.1 (a) 6 / 6

✓ **- 0 pts Correct**

- **2 pts** $$\frac{RT}{F} \times ln \frac{a}{b} = 25mV \times ln10 \times log_{10}\frac{a}{b} = (58 log_{10}\frac{a}{b}) mV$$
- **3 pts** Math mistake.
- **4 pts** $$[F^-]_i = 10mM$$ is incorrect. Extracellular corresponds to space outside the neuron.
- **6 pts** No attempt.

2.2 (b) 7 / 7

✓ **- 0 pts Correct**

- **1 pts** $$V_{pp}$$ is always positive.
- **3 pts** Incorrect $$V_{pp}$$.
- **5 pts** Incorrect $$E_{Na^+}$$.

2.3 (c) 4 / 8

- **0 pts** Correct
- **1 pts** Incorrect final value. (Math mistake)
- **2 pts** Did not state the Goldman equation.
- **2 pts** Correct verbal reasoning. Did not find the relationship between $$P_{F}$$ and $$P_{Na}$$.
- **3 pts** Incomplete justification. Argument still based on known quantities.

✓ **- 4 pts Goldman equation is incorrect.**

- **8 pts** Did not attempt.

2.4 (d) 0 / 4

- **0 pts** Correct
- **1 pts** Math mistake.
- **2 pts** Goldman equation is incorrect.
- **3 pts** Incorrect/insufficient reasoning.

✓ **- 4 pts Incorrect answer.**

1 Must take [F]_i in numerator

2.5 (e)(i) 4 / 4

✓ **- 0 pts Correct**

- **2 pts** Did not mention that resting potential is **LESS** than the equilibrium potential.
- **3 pts** Insufficient reasoning.
- **4 pts** Incorrect or no attempt

2.6 (e)(ii) 4 / 4

✓ **- 0 pts Correct**

- **2 pts** Incorrect direction or no mention of diffusion force.
- **2 pts** Incorrect direction or no mention of drift force.
- **3 pts** Insufficient reasoning.
- **4 pts** Incorrect.

2.7 (e)(iii) 2 / 2

✓ **- 0 pts Correct**

- **1 pts** Insufficient reasoning.
- **2 pts** Incorrect

QUESTION 3

Problem 3 35 pts

3.1 (a) 5 / 5

✓ **- 0 pts Correct**

- **2 pts** Did not mention poisson distribution
- **3 pts** Incorrect parameter for poisson distribution

3.2 (b) 5 / 5

✓ **- 0 pts Correct**

- **3 pts** Incorrect parameter for Poisson distribution
- **2 pts** Incorrect probability value for the event

3.3 (c)(i) **15 / 15**

- ✓ **- 0 pts** **Correct**
- **- 5 pts** Incorrect event for the numerator
- **- 2 pts** Incorrect poisson parameter for N(t)
- **- 2 pts** Incorrect poisson parameter for N(2) - N(t)

3.4 (c)(ii) **0 / 5**

- **- 0 pts** Correct
- **- 2 pts** Incorrect derivative
- ✓ **- 5 pts** **Not attempted**

3.5 (c)(iii) **4 / 5**

- **- 0 pts** Correct
- ✓ **- 1 pts** **Incorrect expected value**
- **- 5 pts** Not attempted

QUESTION 4

4 Bonus **0 / 7**

- **- 0 pts** Correct
- ✓ **- 7 pts** **Incorrect**

1. **True/False and Short Answer.** (30 points)

(a) (10 points) True / False. Answer whether these questions are true or false. Each correct answer is worth +2 point. In this question, you do not need to justify your answers, but we may award partial credit on incorrect answers based on justification.

i. A squid axon neuron, like the one we studied in lecture, is capable of firing over 700 spikes per second.

True

ii. Ligand-gated ion channels, which open when neurotransmitters bind to them at dendrites, are the key type of ion channel that open quickly in a positive feedback loop, leading to action potential generation.

False, voltage-gated ion channels regulated by membrane potential lead to action potential generation

iii. During action potential generation, K^+ voltage-gated ion channels open, leading to an increase in membrane potential, and then enter the inactive state.

False, Na+ voltage-gated ion channels open before inactivating

iv. Consider a neuron with multiple ion species (e.g., Na^+, K^+, Cl^-, Ca^{+2}). For this neuron, all of these ion species can move from inside to outside the cell and vice versa. At steady state, the K^+ ions must always have equal and opposite drift and diffusion currents.

False, depending on the drift current from how these species are concentrated, we could have that drift and diffusion are the same direction

v. Myelination increases membrane capacitance and reduces axial resistance, overall increasing action potential speed.

False, action potential speed is increased but myelination decreases membrane capacitance

1.1 (a) **8 / 10**

- \+ **2 pts** i correct
- ✓ + **2 pts** **ii correct**
- ✓ + **2 pts** **iii correct**
- ✓ + **2 pts** **iv correct**
- ✓ + **2 pts** **v correct**
- \+ **1 pts** i partial credit
- \+ **1 pts** ii partial credit
- \+ **1 pts** iii partial credit
- \+ **1 pts** iv partial credit
- \+ **1 pts** v partial credit
- \+ **0 pts** None correct

(b) (4 points) Explain why the inactive state of Na^+ voltage-gated ion channels is critical for signaling in the nervous system. Answer in no more than 4 sentences.

The inactive state of Na+ arises from the refractory period, which is responsible for unidirectional signaling in the nervous system. Without inactivation, neuron communication would encounter major problems.

(c) (4 points) When an Na^+ ion passes through a sodium ion channel, it sheds a shell of waters around it to bind to a higher energy bonding site. Why does the Na+ ion flow through the ion channel, as opposed to "getting stuck" (i.e., maintaining its bond) at the bonding site? Answer in no more than 4 sentences.

First, the cations in the channel electrostatically attract the negative charge on oxygen as the H_2O bonded to Na+ is dipolar. Then, the Na+ is small enough compared to K+ that it can fit through the ion channel. These two properties prevent it from getting stuck.

1.2 (b) **4 / 4**

✓ **+ 4 pts** **Correct, leads to unidirectional signaling.**

+ 4 pts Correct, prevents Na+ from flowing inside the cell, enabling the action potential voltage to decrease back to a resting state when K+ ion channels open. Note, it was not enough to say it aided in repolarization, but the particular dynamics of how the inactive state aided needed to be described here to get full points.

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+ 2 pts A statement that it limits the rate of signaling in the nervous system, but without an explicit statement why limiting the firing rate is beneficial.

+ 1 pts Description of what the inactivate state is (i.e., prevents a channel from staying open indefinitely) but not why it is critical for nervous system signaling. Remember K+ voltage gated ion channels don't have an inactivate state!

+ 1 pts Incorrect statement that it helps with depolarization (actually, it helps with hyperpolarization), or that it helps with concentration regulation (that's the job of the ion pumps; if you stated that the voltage would not decrease to a level that the voltage gated ion channels would be closed, you received rubric item +2).

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1.3 (c) **2.5 / 4**

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+ **0 pts** No answer

(d) (12 points) You are studying the Porg species in a galaxy far, far away. The Porg is a species capable of flying. You find a Porg neuron that is perfectly modeled by a Poisson process, and is modulated by how high the Porg flies. The Porg neuron's firing rate, λ, is related to the altitude (flying height) of the Porg (in units of feet), h, according to the following tuning curve:

$$\lambda = \frac{1}{10}h.$$

Answer the following questions about this Porg neuron:

i. (3 points) The Porg flies from a height of 10 feet to 100 feet. During this flight, are the inter-spike intervals of the Porg neuron exponentially distributed? Justify your answer in no more than 2 sentences.

No, since λ is not constant, we know that this is an inhomogenerous Poisson Process and will not be exponentially distributed for ISIs.

ii. (3 points) You find that the Porg has very precise flight capabilities, and can hover at the exact same height. You record from the Porg neuron as the Porg is hovering at the same height for a minute. During this time, are the inter-spike intervals of the Porg neuron exponentially distributed? Justify your answer in no more than 2 sentences.

Yes, for this specific interval the firing rate is constant. This independent increment will then be a homogenous Poisson Process which we can model through exponential ISIs.

1.4 (d)(i) **3 / 3**

✓ **+ 3 pts** Correct, not exponential because it's an inhomogeneous Poisson process

+ 0 pts Incorrect.

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Yes, for this specific interval the firing rate is constant. This independent increment will then be a homogenous Poisson Process which we can model through exponential ISIs.

1.5 (d)(ii) **3 / 3**

✓ **+ 3 pts** Correct

+ 2 pts Statement that the firing rate is constant, but no statement on if this implies exponential ISIs or incorrect statement that the ISIs are not exponential

+ 0 pts Incorrect

iii. (6 points) You are able to wirelessly record from this neuron as the Porg flies. You want to use the neuron to decode the altitude of the Porg by estimating the firing rate and computing $h = 10\lambda$. To estimate λ, you count the number of spikes in a 1 second window. Of the possible three choices, when would your decode of the Porg's altitude (height) be most accurate? Choose one of these three, justifying your answer in no more than 4 sentences.

A. It doesn't matter; the Porg neuron has the same accuracy across all altitudes.

(B.) When the Porg is at a height of 10 feet.

C. When the Porg is at a height of 100 feet.

We know from prior that $\lambda = \frac{1}{10}h$. Noise is an important factor to consider here, and the standard deviation for the noise distribution is dependent on the mean for a Poisson process. Therefore, we will want to minimize our firing rate to minimize the noise associated to get the most accurate height.

1.6 (d)(iii) 6 / 6

✓ **+ 6 pts** **Correct, with a justification that the variance of the spike counts will be lower since the spike counts are Poisson distributed**

+ 4 pts Correct, but without explicit justification that there are lower variance spike counts

+ 2 pts Incorrect, with the argument that having more spikes gives a more robust estimate or getting the variance distribution wrong.

+ 2 pts Incorrect, with an argument for why the neuron is similarly accurate at 10ft and 100ft; the most common one is that the tuning function is injective, but this ignores firing rate variance.

+ 0 pts Incorrect without justification, or no answer

2. **Membrane Potential.** (35 points)

The US Space Force is studying an alien neuron in one of the labs at Area 51. They observe that the membrane of this neuron is only permeable to Na^+ and F^-. Intrigued, they get their best team, Alice and Bob, to analyse the neuron further. Alice is an expert in measuring the chemical properties of neurons and Bob's expertise is in measuring their electrical properties.

(a) (6 points) Alice's experiments take more time than Bob's. By the time she finds that the extracellular steady-state concentration of F^- is $10mM$, Bob has already measured the equilibrium potential of F^- and found that it is $116mV$. Alice determines she does not need to spend time measuring the cytoplasmic (i.e., inside the neuron) concentration of F^-. What is the cytoplasmic concentration of F^-?

Hint: You should use the Nernst equation, considering RT/F at room temperature (25° C)

$$E_X = \frac{RT}{zF}\log_{10}\frac{[X]_o}{[X]_i} = \frac{58mV}{z}\log_{10}\frac{[X]_o}{[X]_i}$$

The valence, z, of an ion with negative charge is also negative.

$E_{F^-} = 116$

$116 = \frac{58}{-1}\log\frac{10}{[F^-]_i}$

$-2 = \log\frac{10}{[F^-]_i}$

$.01 = \frac{10}{[F^-]_i}$

$[F^-]_i = 1000mM$

2.1 (a) **6 / 6**

✓ **- 0 pts** **Correct**

- 2 pts $$\frac{RT}{F} \times ln \frac{a}{b} = 25mV \times ln10 \times log_{10}\frac{a}{b} = (58 log_{10}\frac{a}{b}) mV$$

- 3 pts Math mistake.

- 4 pts $$[F^-]_i = 10mM$$ is incorrect. Extracellular corresponds to space outside the neuron.

- 6 pts No attempt.

(b) (7 points) It is faster for Alice to measure a ratio of the steady-state ion concentrations across the membrane. Doing this for Na^+ ions, she finds that they are 10 times more concentrated inside the neuron than outside it. What is the maximum peak-to-peak voltage of the action potential that this neuron can fire?

$$E_{Na^+} = \frac{58}{1} \cdot \log \frac{[a]_o}{[10a]_i}$$

$$= -58 \text{ mV}$$

Peak-to-Peak = 116 + 58 = 174 mV

2.2 (b) **7 / 7**

- ✓ **- 0 pts** **Correct**
- **- 1 pts** V_{pp} is always positive.
- **- 3 pts** Incorrect V_{pp}.
- **- 5 pts** Incorrect E_{Na^+}.

(c) (8 points) Bob measured the resting membrane potential of this neuron and he finds that it is $100mV$. On noticing that this is very close to the equilibrium potential of F^-, Bob quickly concludes that the membrane of the neuron is certainly more permeable to F^- than Na^+. Given just the observations up to this point, Alice argues that there still is a possibility that Bob could be wrong. Show a mathematical justification for why Alice is rightfully cautious.

Hint: You should use the Goldman equation to find a relationship between P_F, the permeability for F^-, and P_{Na}, the permeability for Na^+.

$$E_M = \frac{RT}{F}\ln\left(\frac{P_F[F^-]_o + P_{Na}[Na^+]_o}{P_F[F^-]_i + P_{Na}[Na^+]_i}\right)$$

$$100 = 25\ln\left(\frac{P_F[F^-]_o + P_{Na}[Na^+]_o}{P_F[F^-]_i + P_{Na}[Na^+]_i}\right)$$

$$54.6 = \frac{P_F[F^-]_o + P_{Na}[Na^+]_o}{P_F[F^-]_i + P_{Na}[Na^+]_i}$$

$$54.6 P_F[F^-]_i - P_F[F^-]_o = P_{Na}[Na^+]_o - 54.6 P_{Na}[Na^+]_i$$

$$P_F\left(54.6[F^-]_i - [F^-]_o\right) = P_{Na}\left([Na^+]_o - 54.6[Na^+]_i\right)$$

$$\frac{P_F}{P_{Na}} = \frac{[Na^+]_o - 54.6[Na^+]_i}{54.6[F^-]_i - [F^-]_o}$$

$\therefore$ If $[F^-]_o$ or $[F^-]_i$ are too big, then $P_F / P_{Na} < 1$ and the permeability would not have to be more for F^-

2.3 (c) **4 / 8**

- **- 0 pts** Correct
- **- 1 pts** Incorrect final value. (Math mistake)
- **- 2 pts** Did not state the Goldman equation.
- **- 2 pts** Correct verbal reasoning. Did not find the relationship between $$P_{F}$$ and $$P_{Na}$$.
- **- 3 pts** Incomplete justification. Argument still based on known quantities.
- ✓ **- 4 pts** **Goldman equation is incorrect.**
- **- 8 pts** Did not attempt.

(d) (4 points) Wary of Bob's hasty conclusion, Alice decides to gather more information to correctly estimate the permeability of the membrane. She now measures the extracellular steady-state concentration of Na^+ and finds that it is $1mM$. Is the membrane more permeable to F^-?

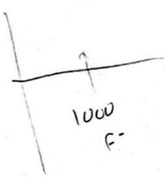
$[Na^+]_o = 1$, $[Na^+]_i = 10$ $[F^-]_o = 10$, $[F^-]_i = 1000$

From the previous equation

$$\frac{P_F}{P_{Na}} = \frac{1 - 54.6(10)}{54.6(1000) - 10}$$

→ We can clearly see the membrane is <u>less</u> permeable to F^-

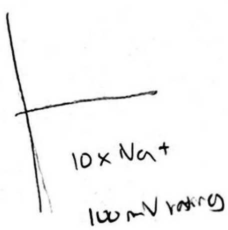
(e) (10 points) Finally, Alice concludes the experiment and states the following facts about the steady-state of the neuron. For each of these true statements, provide a brief explanation (no more than 4 sentences each) for why it is true.



i. (4 points) The chemical diffusion force on F^- is greater than the electric drift force on it.

Due to such a large concentration of $[F^-]_i$ relative to $[F^-]_o$, the diffusion force will be greater than the drift current opposing this diffusion. We can see equilibrium potential of F^- is greater than equilibrium potential of the membrane, indicating the chemical diffusion force is larger.

ii. (4 points) The net electro-chemical force on Na^+ pushes it out of the cell.



The drift current pushes Na+ out as the resting potential is a positive 100 mV. The diffusion current pushes Na+ out as there is 10x more inside than outside.

iii. (2 points) The alien neuron has ion-pumps that maintain the ion concentrations. They pump Na^+ and F^- into the cell.

We established prior both F- and Na+ are pushed out of the cell. In order to allow for action potentials, steady state is required whereby the ion pumps put energy into the system to maintain concentration gradients. These pump Na+ and F- against their electro-chemical forces.

2.4 (d) **0 / 4**

- **- 0 pts** Correct
- **- 1 pts** Math mistake.
- **- 2 pts** Goldman equation is incorrect.
- **- 3 pts** Incorrect/insufficient reasoning.
- ✓ **- 4 pts Incorrect answer.**

1 Must take [F]_i in numerator

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From the previous equation

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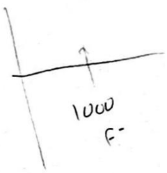

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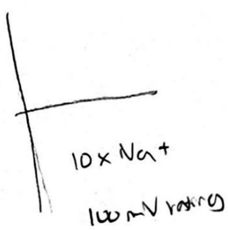

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iii. (2 points) The alien neuron has ion-pumps that maintain the ion concentrations. They pump Na^+ and F^- into the cell.

We established prior both F- and Na+ are pushed out of the cell. In order to allow for action potentials, steady state is required whereby the ion pumps put energy into the system to maintain concentration gradients. These pump Na+ and F- against their electro-chemical forces.

2.5 (e)(i) **4 / 4**

- ✓ **- 0 pts** **Correct**
- **- 2 pts** Did not mention that resting potential is **LESS** than the equilibrium potential.
- **- 3 pts** Insufficient reasoning.
- **- 4 pts** Incorrect or no attempt

(d) (4 points) Wary of Bob's hasty conclusion, Alice decides to gather more information to correctly estimate the permeability of the membrane. She now measures the extracellular steady-state concentration of Na^+ and finds that it is $1mM$. Is the membrane more permeable to F^-?

$[Na^+]_o = 1$, $[Na^+]_i = 10$ $[F^-]_o = 10$, $[F^-]_i = 1000$

From the previous equation

$$\frac{P_F}{P_{Na}} = \frac{1 - 54.6(10)}{54.6(1000) - 10}$$

→ We can clearly see the membrane is less permeable to F^-

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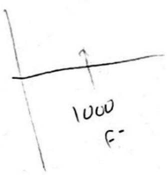

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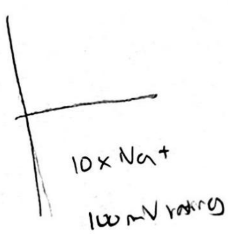

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2.6 (e)(ii) **4 / 4**

- ✓ **- 0 pts** **Correct**
- **- 2 pts** Incorrect direction or no mention of diffusion force.
- **- 2 pts** Incorrect direction or no mention of drift force.
- **- 3 pts** Insufficient reasoning.
- **- 4 pts** Incorrect.

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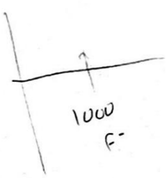
$$\frac{P_F}{P_{Na}} = \frac{1 - 54.6(10)}{54.6(1000) - 10}$$

→ We can clearly see the membrane is less permeable to F^-

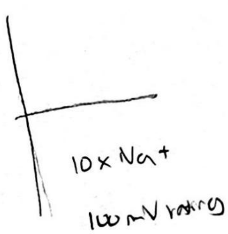
(e) (10 points) Finally, Alice concludes the experiment and states the following facts about the steady-state of the neuron. For each of these true statements, provide a brief explanation (no more than 4 sentences each) for why it is true.



i. (4 points) The chemical diffusion force on F^- is greater than the electric drift force on it.

Due to such a large concentration of $[F^-]_i$ relative to $[F^-]_o$, the diffusion force will be greater than the drift current opposing this diffusion. We can see equilibrium potential of F^- is greater than equilibrium potential of the membrane, indicating the chemical diffusion force is larger.

ii. (4 points) The net electro-chemical force on Na^+ pushes it out of the cell.



The drift current pushes Na+ out as the resting potential is a positive 100 mV. The diffusion current pushes Na+ out as there is 10x more inside than outside.

iii. (2 points) The alien neuron has ion-pumps that maintain the ion concentrations. They pump Na^+ and F^- into the cell.

We established prior both F- and Na+ are pushed out of the cell. In order to allow for action potentials, steady state is required whereby the ion pumps put energy into the system to maintain concentration gradients. These pump Na+ and F- against their electro-chemical forces.

2.7 (e)(iii) **2 / 2**

- ✓ **- 0 pts** Correct
- **- 1 pts** Insufficient reasoning.
- **- 2 pts** Incorrect

(a) (5 points) Find the distribution of the number of spikes fired on a given day.

Hint: If you think of 10am as the starting time of the process, then this part of the problem is asking you to compute the distribution of $N(8)$.

For an Inhomogenous PP

$$N(8) \sim PP\left(\int_0^8 \lambda(t)\, dt \right)$$

$$\sim \text{Poisson}\left(\tfrac{1}{2}(2)(4) + (2)(4) + \tfrac{1}{2}(2)(2) + (2)(2) + \tfrac{1}{2}(2)(4) + \tfrac{1}{2}(2)(2) \right)$$

$$\boxed{\sim \text{Poisson}(24)}$$

(b) (5 points) Find the probability that no spikes are fired until 12pm. Again, you can assume without loss of generality that the process starts at 10am. You may leave your answer in terms of e.

→ In first two hours, no spikes are fired

$$P(N(2) = 0) \sim \text{Poisson}(4)$$

$$= \frac{(4)^0 e^{-4}}{0!} = \boxed{e^{-4}}$$

3.1 (a) **5 / 5**

- ✓ **- 0 pts** Correct
- **- 2 pts** Did not mention poisson distribution
- **- 3 pts** Incorrect parameter for poisson distribution

(a) (5 points) Find the distribution of the number of spikes fired on a given day.

Hint: If you think of 10am as the starting time of the process, then this part of the problem is asking you to compute the distribution of $N(8)$.

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$$N(8) \sim PP\left(\int_0^8 \lambda(t)\, dt \right)$$

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$$\boxed{\sim \text{Poisson}(24)}$$

(b) (5 points) Find the probability that no spikes are fired until 12pm. Again, you can assume without loss of generality that the process starts at 10am. You may leave your answer in terms of e.

→ In first two hours, no spikes are fired

$$P(N(2) = 0) \sim \text{Poisson}(4)$$

$$= \frac{(4)^0 e^{-4}}{0!} = \boxed{e^{-4}}$$

3.2 (b) **5 / 5**

- ✓ **- 0 pts** Correct
- **- 3 pts** Incorrect parameter for Poisson distribution
- **- 2 pts** Incorrect probability value for the event

I cont'd
↑

(c) (25 points) Assume that exactly two spikes have been fired from 10am-12pm. In this part of the problem, we will walk you through the steps for finding the expected firing time of the second spike. For this part of the problem too, you can assume without loss of generality that the process starts at 10am. You may leave your answer in terms of e.

i. (15 points) Let's define the random variable T_2 as the firing time of the second spike with units of hours. Also, let's define the event A that exactly two spikes have been fired from 10am-12pm. With these definitions, compute the cumulative distribution function (CDF) of T_2 conditioned on the event A i.e find $F_{T_2|A}(t|A)$. (sum of probabilities)

Hint: $F_{T_2|A}(t|A) = P(T_2 \leq t|A)$, where $0 \leq t \leq 2$.

exponential CDF

$F(x) = 1 - e^{-\lambda x}$

$P(A) = P(N(2) = 2) = \text{Poisson}(4)$

$= \frac{4^2 e^{-4}}{2!} = 8e^{-4}$

$F_{T_2|A}(T_2 = t | A)$

$= \frac{f(T_2 = t, A)}{f(A)}$

$= \frac{f(A | T_2 = t) f(T_2 = t)}{f(A)}$

2 spikes up to t ; 0 spikes from t to 2

$\frac{(2t)^2 e^{-2t}}{2!} \times \frac{(8-2t)^0 e^{-(8-2t)}}{0!}$

by independent increments

3.3 (c)(i) **15 / 15**

- ✓ **- 0 pts** **Correct**
- **- 5 pts** Incorrect event for the numerator
- **- 2 pts** Incorrect poisson parameter for N(t)
- **- 2 pts** Incorrect poisson parameter for N(2) - N(t)

ii. (5 points) Use the conditional CDF computed in the previous part to compute the probability density function (PDF) of T_2 conditioned on the event A i.e. find $f_{T_2|A}(t|A)$.

Hint: $f_{T_2|A}(t|A) = \frac{d}{dt}F_{T_2|A}(t|A)$, *where* $0 \leq t \leq 2$.

iii. (5 points) Use the conditional PDF to compute $E[T_2|A]$ and then use it to compute the expected firing time of the second spike conditioned on event A.

Hint: $E[\text{firing time of second spike}|A] = 10am + E[T_2|A]$.

$E[T_2 | A]$

$= \int_0^\infty t f(t \mid A) dt$

$= \int_0^\infty t \frac{f(A \mid t) f(t)}{f(A)} dt$

$= \int_0^\infty t \cdot \frac{(2t)^2 e^{-2t}}{2!} \cdot \frac{e^{-(8-2t)}}{0!}$

3.4 (c)(ii) **0 / 5**

- **- 0 pts** Correct
- **- 2 pts** Incorrect derivative
- ✓ **- 5 pts Not attempted**

ii. (5 points) Use the conditional CDF computed in the previous part to compute the probability density function (PDF) of T_2 conditioned on the event A i.e. find $f_{T_2|A}(t|A)$.

Hint: $f_{T_2|A}(t|A) = \frac{d}{dt} F_{T_2|A}(t|A)$, *where* $0 \leq t \leq 2$.

iii. (5 points) Use the conditional PDF to compute $E[T_2|A]$ and then use it to compute the expected firing time of the second spike conditioned on event A.

Hint: E[firing time of second spike$|A] = 10am + E[T_2|A]$.

$E[T_2 | A]$

$= \int_0^\infty t f(t | A) dt$

$= \int_0^\infty t \frac{f(A | t) f(t)}{f(A)} dt$

$= \int_0^\infty t \cdot \frac{(2t)^2 e^{-2t}}{2!} \cdot \frac{e^{-(8-2t)}}{0!}$

3.5 (c)(iii) **4 / 5**

- **- 0 pts** Correct
- ✓ **- 1 pts Incorrect expected value**
- **- 5 pts** Not attempted

4 Bonus **0 / 7**

- **- 0 pts** Correct
- ✓ **- 7 pts** **Incorrect**