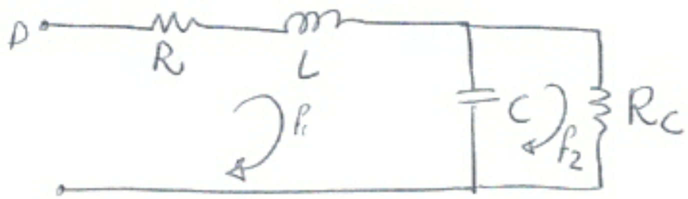
1-1-



KVL @ loop1 : $p - Rf_1 - L\frac{df_1}{dt} - \frac{1}{C}\int (f_1 - f_2)\,dt = 0$

KVL @ loop2 : $\frac{-1}{C}\int (f_2 - f_1)\,dt - f_2 R_C = 0 \Rightarrow \frac{1}{C}\int f_1\,dt = f_2 R_C + \frac{1}{C}\int f_2\,dt$

$\Rightarrow f_1 = C R_C f_2' + f_2 \rightsquigarrow$ replace f_1 in loop1 $\rightsquigarrow$

$$\rightsquigarrow p - RCR_c f_2' - Rf_2 - LCR_c f_2'' - Lf_2' - f_2 R_c = 0$$

$$\rightsquigarrow (LCR_c)\frac{d^2 f_2}{dt^2} + (L + RCR_c)\frac{df_2}{dt} + (R_c + R)f_2 = p$$

$(f_2 = f)$

1-2- $(LCR_c)s^2 F(s) + (L + RCR_c)sF(s) + (R_c + R)F(s) = P(s)$

$$[(LCR_c)s^2 + (L + RCR_c)s + (R_c + R)]F(s) = P(s)$$

$$T = \frac{F}{P} = \frac{1}{(LCR_c)s^2 + (L + RCR_c)s + (R_c + R)}$$

1-3- $T = \frac{1}{(2C)s^2 + (1+2RC)s + (2+R)} = \frac{1}{2C} \times \frac{1}{s^2 + \left(\frac{1+2RC}{2C}\right)s + \left(\frac{2+R}{2C}\right)}$

$t_r = \frac{1.8}{\omega_n} \leq 0.6 \rightsquigarrow \omega_n \geq 3 \rightsquigarrow \sqrt{\frac{2+R}{2C}} \geq 3$

$t_s = \frac{4.6}{\xi\omega_n} \leq 4.6 \rightsquigarrow \xi\omega_n \geq 1 \rightsquigarrow \frac{1+2RC}{4C} \geq 1$

R, C are positive values $\Rightarrow \begin{cases} 0 \leq R \\ 0 \leq C \leq \frac{R+2}{18} \end{cases}$

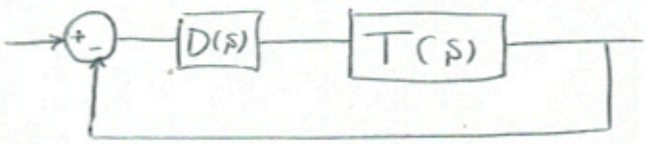
1-4- $T = \dfrac{1}{s^2+5s+6}$, output is $f \rightsquigarrow$ we are looking for $f(t)$

$$F(s) = T(s)\times p(s) = \frac{1}{s^2+5s+6} \times \frac{2}{s} = \frac{2}{s(s+3)(s+2)}$$

$$\Rightarrow F(s) = \frac{\frac{1}{3}}{s} + \frac{-1}{s+2} + \frac{\frac{2}{3}}{s+3}$$

$$\Rightarrow f(t) = \left(\frac{1}{3} - e^{-2t} + \frac{2}{3}e^{-3t}\right)u(t)$$

1-5-



$$T_{cl} = \frac{DT}{1+DT} = \frac{D(s)}{s^2+5s+6+D(s)}$$

Let's see if we can meet the spesification with a proportional controller $\longrightarrow D = k_p$

$$T_{cl} = \frac{k_p}{s^2+5s+6+k_p} \rightsquigarrow \text{stability check} \rightsquigarrow \left.\begin{array}{lll} s^2 & 1 & 6+k_p \\ s^1 & 5 & 0 \\ s^0 & 6+k_p & \end{array}\right\} \Rightarrow 6+k_p \geq \\ \rightsquigarrow k_p \geq -6$$

spesifications :

$$t_r \leqslant 0.6 \rightsquigarrow \frac{1.8}{\sqrt{6+k_p}} \leqslant 0.6 \rightsquigarrow k_p \geqslant 3$$

$$t_s \leqslant 4.6 \rightsquigarrow \frac{4.6}{2.5} \leqslant 4.6 \ \checkmark$$

we can satisfy the spesification by a proportional controller with $k_p > 3$

2.1 Take the Routh table: $s^3 + 2s^2 + ks + (2k-4) = 0$

s^3	1	K
s^2	2	$2K-4$
s^1	2	
s^0	$2K-4$	

No sign changes iff $2K - 4 > 0$

$\Rightarrow$ System is stable for $\underline{K > 2}$.

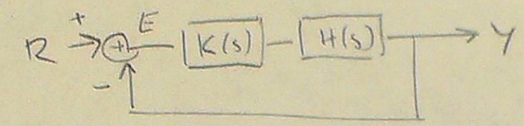
2.2 Denote $H(s) = \frac{Y(s)}{R(s)}$, $E(s) = R(s) - Y(s)$.

Then for $K = 5$, the system is stable (as $K > 2$) so the FVT can be used.

$$e_{ss} = 1 - y_{ss} = 1 - \lim_{s \to 0} s \frac{Y(s)}{R(s)} R(s) \; ; \qquad R(s) = \frac{1}{s}$$

$$\Rightarrow e_{ss} = 1 - \lim_{s \to 0} \frac{\cancel{s}}{\cancel{s}} \frac{3s^2 + 3s + 2}{s^3 + 2s^2 + 5s + 6} = 1 - \frac{2}{6} = \underline{\frac{2}{3}}.$$

2.3



$$\frac{Y(s)}{R(s)} = \frac{KH}{1 + KH} \; ; \quad \frac{E(s)}{R(s)} = \frac{1}{1 + KH}.$$

For steady-state error, $e_{ss} = \lim_{s \to 0} s \frac{E(s)}{R(s)} R(s)$ iff stable; $R(s) = \frac{1}{s}$.

$$\frac{E(s)}{R(s)} = \frac{s^3 + 2s^2 + 5s + 6}{s^3 + 2s^2 + 5s + 6 + K(s)(3s^2 + 3s + 2)}$$

Try $K(s) = \frac{1}{s} K_I \Rightarrow \frac{E(s)}{R(s)} = \frac{s^4 + 2s^3 + 5s^2 + 6s}{s^4 + 2s^3 + 5s^2 + 6s + 3K_I s^2 + 3K_I s + 2K_I}$

Then,

$$e_{ss} = \lim_{s \to 0} \frac{\cancel{s}}{\cancel{s}} \frac{s^4 + 2s^3 + 5s^2 + 6s}{s^4 + 2s^3 + 5s^2 + 6s + 3K_I s^2 + 3K_I s + 2K_I} = 0$$

iff stable.

2.3 (cont.)

Check stability via Routh table:

$$\alpha(s) = s^4 + 2s^3 + (5+3K_I)s^2 + (6+3K_I)s + 2K_I$$

s^4	1	$5+3K_I$	$2K_I$	0
s^3	2	$6+3K_I$	0	0
s^2	$\frac{9}{b}$	$2K_I$	0	
s^1	b	0	0	
s^0	$2K_I$	0		

Can solve directly, or test values. For example, $K_I = 1$ gives

s^4	1	8	2	0
s^3	2	9	0	0
s^2	$\frac{7}{2}$	0	0	
s^1	$\frac{55}{7}$	0		
s^0	2	0		

So $K(s) = \frac{1}{s}$ is a valid choice to meet the specifications.