EE141 Principles of Feedback Control (Winter 2013)
Solutions to Final

Problem 1
(1)
The transfer function between x_1 and u is

$$\frac{x_1(s)}{u(s)} = \frac{2}{s^2 - 1} \tag{1}$$

When a step is applied as input at $t = 0$, $u(s) = 1/s$, we can get the output x_1 in the s domain

$$x_1(s) = \frac{2}{s(s^2 - 1)} \tag{2}$$

In the time domain,

$$x_1(t) = (e^{-t} + e^{t} - 2)u(t), \tag{3}$$

where $u(t)$ is the step function.

(2)
With the controller, the transfer function between x_1 and u becomes

$$\frac{x_1(s)}{u(s)} = \frac{2}{s^2 + 2K_D s + 2K_P - 1} \tag{4}$$

Rise time is 0.9 seconds, $t_r = \frac{1.8}{w_n} = 0.9$, then $w_n = 2$, which means

$$\frac{1.8}{\sqrt{2K_P - 1}} = 0.9 \tag{5}$$

Then we get $K_P = 2.5$. Damping ratio is 1, which means

$$\frac{K_D}{w_n} = 1 \tag{6}$$

Then we get $K_D = 2$.

(3)

$$t_s = \frac{4.6}{K_D} = 2.3s \tag{7}$$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = 0 \tag{8}$$

(4)
For the transfer function,

$$\frac{x_1(s)}{u(s)} = \frac{2}{s^2 + 4s + 4}, \tag{9}$$

we can get the step response in Figure 1,

(5)

$$e_{ss} = \lim_{s\to 0} s\left(-\frac{2}{s^2(s^2 + 4s + 4)} + \frac{1}{s^2}\right) = \infty \tag{10}$$

The system cannot track a ramp.

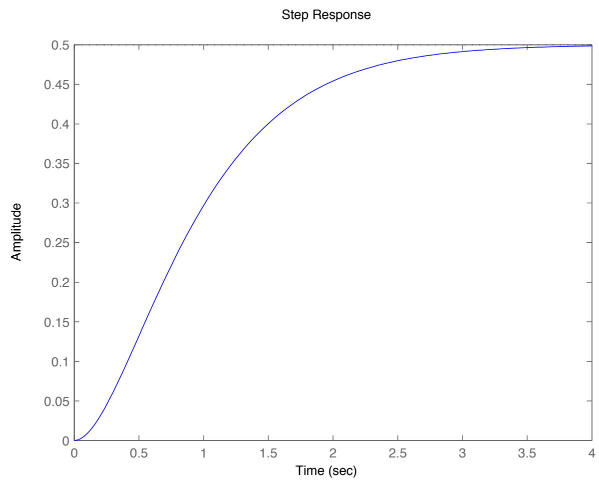

Figure 1: Step response

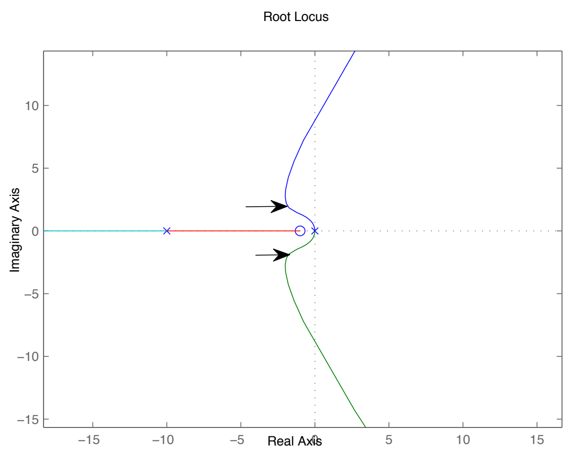

Figure 2: Root locus

Problem 2

$$G(s) = \frac{s+1}{s^2(s+10)^2} \tag{11}$$

(1)
The root locus is shown in Figure 2.
(2)
To reduce the overshoot, we need to reduce the damping ratio ξ. The arrows in Figure 2 point to the closed-loop poles we should choose.

(3)

$$G'(s) = \frac{s+1}{s^2(s+10)} \tag{12}$$

Using Routh's criteria, we can get when $K > 0$, the new system is stable.
(4)

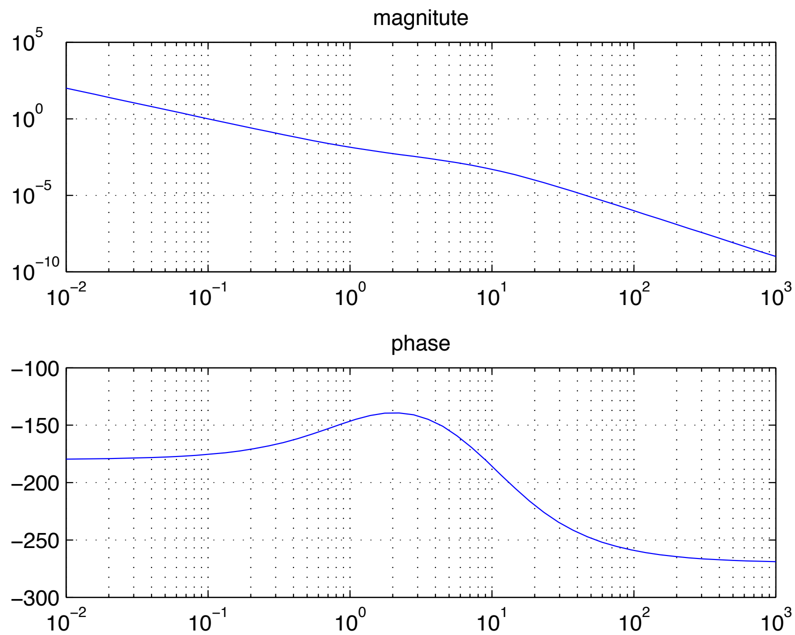

Figure 3: Bode plot of system G.

The Bode plot of G is shown in Figure 3.

(5)
The phase margin is 5 degree. As the phase margin is small, if you said the phase margin is approximate 0, you are right.

(6)
The bode plot is shown in Figure 4. Phase margin is about 20 degree(need not to be accurate). In question 3, we show that the system is stable for any K. In the bode plot, we can see that the phase is never -180 degree, so the system is stable no matter what K we choose. This is in agreement with the result of question 3.

(7)
From the bode plot, we can see the maximum phase margin is about 55 degree(need not to be accurate).

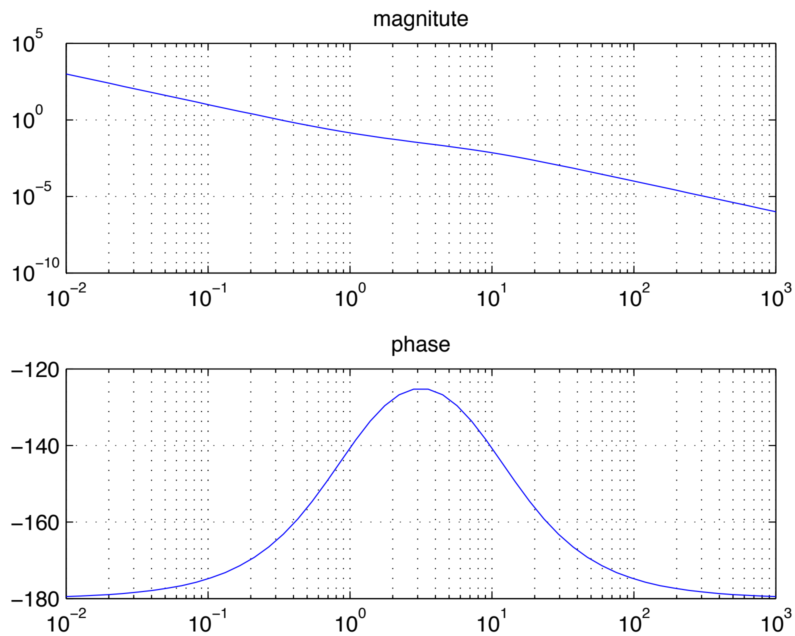

Figure 4: Bode plot of system G′.