EE141 Principles of Feedback Control (Winter 2010)
Solutions to Final

Problem 1
(1)

$$
\begin{aligned}
sX_1 &= -0.4X_1 + 0.2X_2 + 0.02X_3 + U \\
sX_2 &= 0.2X_1 - 0.4X_2 \\
sX_3 &= 0.2X_1 - 0.02X_3
\end{aligned}
$$

Then we can eliminate X_2, X_3 and get

$$\frac{X_1}{U} = \frac{(s+0.4)(s+0.02)}{s^3 + 0.82s^2 + 0.132s + 0.0008}$$

(2)
With Routh Test,

s^3	1	0.132
s^2	0.82	0.0008
s^1	0.131	
s^0	0.0008	

The first column is positive. Thus the system is stable.

(3)
$U = \frac{0.5}{s}$ and since the system is stable. The steady state concentration of insulin in the plasma upon an injection is

$$
\begin{aligned}
&\lim_{s\to 0} sX_1(s) \\
&= \lim_{s\to 0} s\frac{(s+0.4)(s+0.02)}{s^3 + 0.82s^2 + 0.132s + 0.0008}\frac{0.5}{s} \\
&= 5
\end{aligned}
$$

(4)
G_3 has the fastest response due to two most negative poles. Thus G_3 is corresponding to the middle graph.
G_1 has a relative larger steady state response than G_2. Thus G_1 is corresponding to the left graph and G_2 corresponding to the right graph.

(5)

$$H(s) = \frac{1}{s^2 + 0.606s + 0.0036}$$

We can use a PD controller on the feedback path here, then the close-loop transfer function is

$$\frac{H}{1+DH} = \frac{\frac{1}{s^2+0.606s+0.0036}}{1+(k_D s+k_P)\frac{1}{s^2+0.606s+0.0036}}$$
$$= \frac{1}{s^2+(k_D+0.606)s+k_p+0.0036}$$

Since

$$t_s = \frac{4.6}{\sigma} \leq 1 \Rightarrow \sigma \geq 4.6$$
$$t_r = \frac{1.8}{\omega_n} \leq 0.9 \Rightarrow \omega_n \geq 2$$

we have

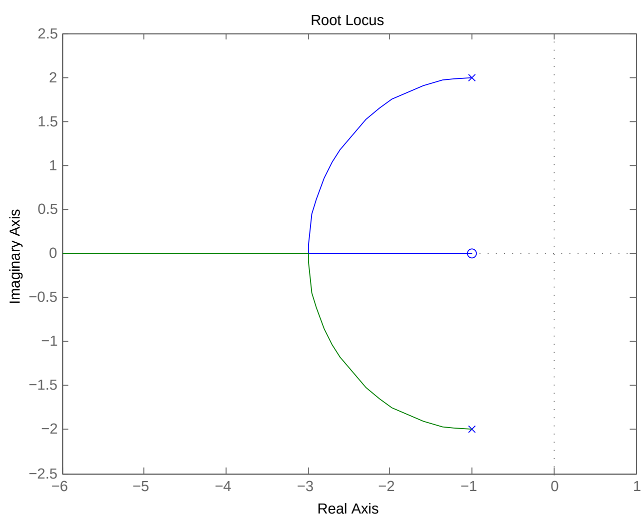
$$k_D + 0.606 = 2\sigma \geq 9.2, \quad k_p + 0.0036 = \omega_n^2 \geq 4$$
$$\Rightarrow k_D \geq 8.594, k_P \geq 3.9964$$

We can pick $k_D = 9, k_P = 25$, such that $\omega_n > 2, \zeta < 1, \sigma > 4.6$. With Routh Test, the system with this PD controller is stable.

Problem 2

(1)

The root locus is



Departure angles at the poles are ±180

Break-in point is −3

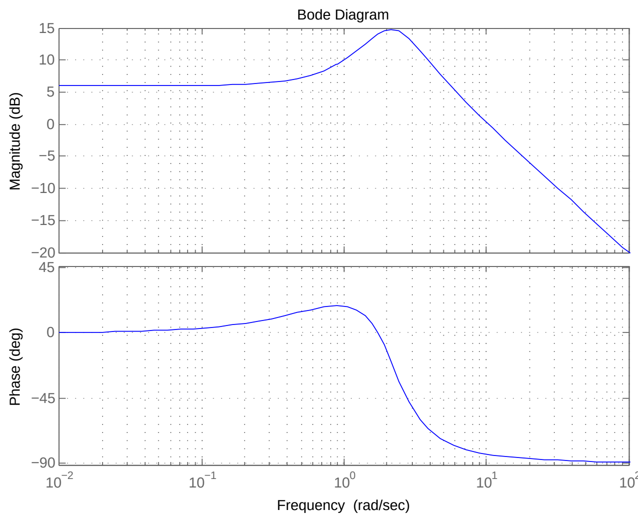
Break-in angles are ±90

(2)
To reduce the settling time, we need to make the real part of the poles as negative as possible. This requires the poles to be placed at the break-in point. Thus

$$1 + K\frac{10(s+1)}{s^2+2s+5} = 0, s = -3$$
$$\Rightarrow K = 0.4$$

(3)
Since the root locus is not touching imaginary axis for any $K > 0$. Thus the bode plot will not touch $phase = -180$, and the gain margin here is infinity.

(4)
The Bode plot is



We can see that we cannot have $phase = -180$ and the gain margin is infinity.

(5)

$$G(s) = \frac{10(s+1)}{s^2+2s+5} = \frac{2(s+1)}{(\frac{s}{\sqrt{5}})^2 + \frac{2}{5}s + 1}$$

The original DC gain is 2. To make the DC gain equal to $20dB = 10$. We need a factor of 5. Also to make the magnitude plot decays at 40 dB/dec for all the frequencies greater than 100 rad/s, we need a pole around 10. Thus the compensator is

$$D(s) = \frac{5}{\frac{s}{10} + 1}$$

With Rooth test the system is stable.

(6)

After the compensator, the close-loop transfer function is

$$\frac{DG}{1+DG} = \frac{\frac{5}{\frac{s}{10}+1}\frac{10(s+1)}{s^2+2s+5}}{1+\frac{5}{\frac{s}{10}+1}\frac{10(s+1)}{s^2+2s+5}}$$
$$= \frac{500(s+1)}{(s+10)(s^2+2s+5)+500(s+1)}$$

With Rooth test the system is stable. Thus the steady state error of the unitary step input is

$$\lim_{s\to 0} sE(s)$$
$$= \lim_{s\to 0} s(1-\frac{500(s+1)}{(s+10)(s^2+2s+5)+500(s+1)})\frac{1}{s}$$
$$= 0.091$$

Thus system cannot track the step input.