Q1. $\tau_1 \dot{T} = -T + U \xrightarrow{\mathcal{L}\{\}} s\tau_1 T = -T + U$

$T(1+s\tau_1) = U$

$\tau_2 \ddot{Z} + \dot{Z} = aT + \cancel{\phi} \xrightarrow{\mathcal{L}\{\}} s^2 \tau_2 Z + sZ = aT$

$$\therefore \quad s(1+s\tau_2)Z = \frac{aU}{1+s\tau_1}$$

$$\therefore \frac{Z}{U} = \frac{a}{s(1+s\tau_1)(1+s\tau_2)} = \frac{10}{s(1+0.1s)(1+0.2s)}$$

2. Closed loop TF: $\frac{G(s)C(s)}{1+G(s)C(s)}$ where $G(s) = \frac{Z}{U}$, $C(s)$ is the controller

a) e_{ss} to a unity parabola $= 0.2$:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\, E(s) = \lim s \cdot \underset{\substack{\uparrow \\ E/\text{input}}}{\frac{1}{1+G(s)C(s)}} \cdot \underset{\substack{\uparrow \\ \text{input}}}{\frac{\cancel{2}A}{s^3}}$$

$$= \lim_{s\to 0} \frac{\cancel{2}A}{s^2} \frac{\cancel{10}\ s(1+0.1s)(1+0.2s)}{s(1+0.1s)(1+0.2s) + 10\,C(s)}$$

choosing a PI controller to control e_{ss} and maintain stability:

$$C(s) = K + \frac{K_I}{s}$$

assuming $A = 1$:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} \frac{1}{s^2} \frac{s(1+0.1s)(1+0.2s)}{s(1+0.1s)(1+0.2s) + 10\left(K + \frac{K_I}{s}\right)}$$

$$= \lim_{s\to 0} \frac{(1+0.1s)(1+0.2s)}{s^2(1+0.1s)(1+0.2s) + 10(sK + K_I)} = \frac{1}{10K_I} = 0.2$$

$\therefore K_I = 0.5$

b) checking stability:

characteristic polynomial:

$$0.02 s^4 + 0.3 s^3 + s^2 + 10Ks + 5 = 0$$

$$s^4 + 15 s^3 + 50 s^2 + 500Ks + 250 = 0$$

applying Routh's criterion:

1	50	250
15	500K	
$50 - \frac{100K}{3}$	250	
$\frac{25[40K^2 - 60K + 9]}{2K-3}$		
250		

$\therefore$ ① $500K > 0 \rightarrow K > 0$

② $50 - \frac{100K}{3} > 0 \rightarrow K < 3/2$

③ $\frac{25[40K^2 - 60K + 9]}{2K-3} > 0 \rightarrow$ $\because$ from ②: $K < 3/2$ $\therefore 2K - 3 < 0$

$\therefore$ $40K^2 - 60K + 9 < 0$

$\therefore$

$\therefore$ $0.1691 < K < 1.331$

choose $K = 1$

$\therefore$ $C(s) = 1 + \frac{0.5}{s}$

Problem 2

(1)

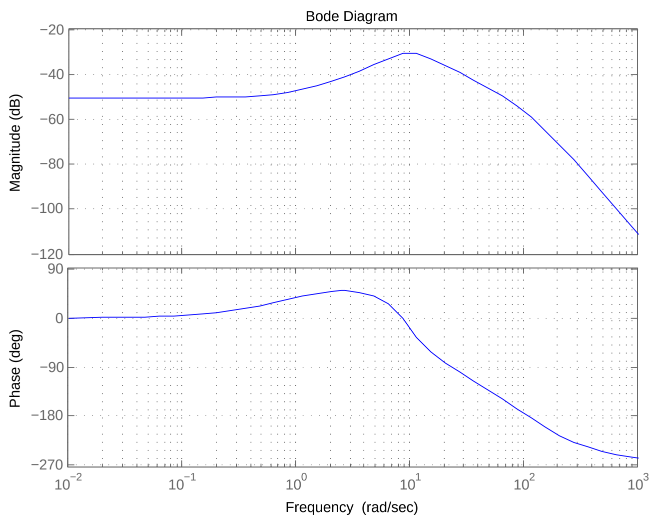
$$H(s) = Y(s)/U(s) = \frac{\frac{C}{1+CD}G}{1 + \frac{C}{1+CD}G} = \frac{CG}{1 + CD + CG}$$

(2)

When D(s)=0 and C(s)=3000, the open-loop transfer function is

$$\frac{CG}{1+CD} = \frac{3000(s+1)}{(s^2+10s+100)(s+100)^2} = 3 \times 10^{-3} \frac{(s+1)}{((s/10)^2 + s/10 + 1)(s/100+1)^2}$$

The bode plot is:



(3)

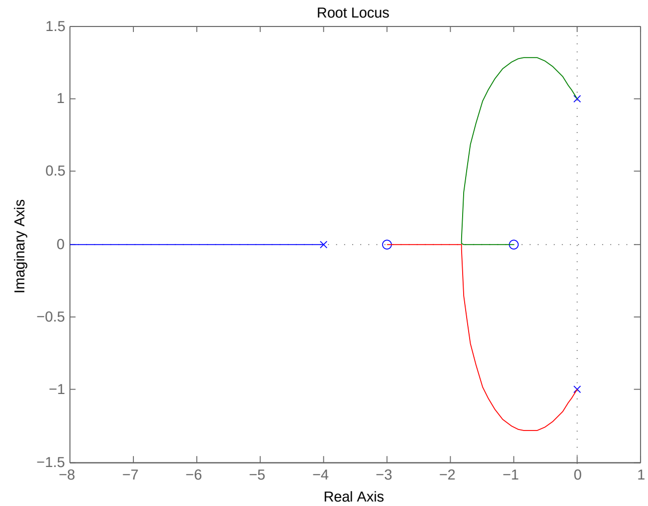
When the gain at $\omega = 0.01$rad/s is changed from -50 dB to 10 dB, the bode plot is shifted upwards by 60 dB. From the bode plot in part(2), we can see that in the phase plot the curve crosses $-180°$ at approximately $\omega = 100$. The gain margin at $\omega = 100$ is less than 60 dB. Therefore, the system will not remain stable in this case.

Problem 3

(1)

$$G(s) = \frac{s^3 + 4s^2 + s + 4}{s^3 + 4s^2 + s + 4 + k(s^2 + 4s + 3)}$$
$$= \frac{1}{1 + k\frac{(s+1)(s+3)}{(s+4)(s^2+1)}}$$

Set $1 + k\frac{(s+1)(s+3)}{(s+4)(s^2+1)} = 0$, the root locus is



The departure angle at the poles are

$$\begin{aligned}\phi_{dep} &= 45^\circ + tan^{-1}(1/3) - (90^\circ + tan^{-1}(1/4)) - 180^\circ \\ &= 45^\circ + 18.5^\circ - (90^\circ + 14^\circ) - 180^\circ \\ &= -220.5^\circ\end{aligned}$$

By symmetry, the departure angels at the other pole is 220.5°.

If applying Rule#6, we can find the break-in point by solving (not required in exam)

$$a(s)\frac{db(s)}{ds} = b(s)\frac{da(s)}{ds}$$
$$(s^3 + 4s^2 + s + 4)(2s + 4) = (s^2 + 4s + 3)(3s^2 + 8s + 1)$$
$$s = -1.8$$

(2)

To reduce the settling time, we need to make the real part of the poles as negative as possible. This requires the poles to be placed at the break-in point. At this point the poles as purely real, hence there will be no overshoot.

(3)

From the plot of root locus, for $k > 0$, the poles are all at LHP, and the poles are on the real axis only when $k = 0$. For $k < 0$, there will be pole(s) in RHP. Thus ($k < 0$ not required in exam)

k	stability	phase margin
positive	stable	positive
zero	neutrally stable	zero
negative	unstable	negative