ECE 141 Midterm

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0. I have read and understood UCLA Student Conduct Code, items 102.01 titled Academic Dishonesty and 102.02 titled other forms of Dishonesty. MikeQu

Problem 1.

1. Persons P_1, P_2

$y_1(t)$, $y_2(t)$ differentiable.

Distance between $P_1P_2 = u(t)$.

$$\begin{cases} \frac{d}{dt} y_1(t) = u(t) - k_1 y_1(t) \\ \frac{d}{dt} y_2(t) = y_1(t) - k_2 y_2(t) \end{cases}$$

2. Input: $u(t)$

$$Y_2(s) = \frac{1}{s^2 + \alpha s + \beta}.$$

$$\begin{cases} sY_1(s) - \cancel{y_1(0)} = U(s) - k_1 Y_1(s) \\ sY_2(s) - \cancel{y_2(0)} = Y_1(s) - k_2 Y_2(s) \end{cases}$$

$$(s+k_2) Y_2(s) = Y_1(s)$$

$$(s+k_1) Y_1(s) = U(s).$$

$$\frac{s+k_2}{s^2+\alpha s+\beta} = \frac{U(s)}{s+k_1} \leftarrow \begin{array}{l} u(t)=\delta(t) \\ U(s)=\text{constant.} \end{array}$$

$$(s+k_1)(s+k_2) = s^2+\alpha s+\beta.$$

$\alpha = k_1+k_2$

$\beta = k_1 k_2$

$$\alpha^2 = k_1^2 + 2k_1k_2 + k_2^2$$
$$\alpha^2 - 4\beta = k_1^2 - 2k_1k_2 + k_2^2$$
$$\sqrt{\alpha^2-4\beta} = k_1 - k_2$$

$$k_1 = \frac{\alpha+\sqrt{\alpha^2-4\beta}}{2}$$
$$k_2 = \frac{\alpha-\sqrt{\alpha^2-4\beta}}{2}$$

3.

$$u(t) = \begin{cases} 1 & \forall t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$U(s) = \frac{1}{s}.$$

$$\begin{cases} (s+k_2)\,Y_2(s) = Y_1(s) \\ (s+k_1)\,Y_1(s) = U(s). \end{cases}$$

a) $$Y_2(s) = \frac{U(s)}{(s+k_1)} \cdot \frac{1}{(s+k_2)}$$
$$= \frac{1}{s(s+k_1)(s+k_2)}$$

b) $$Y_2(s) = \frac{A}{s} + \frac{B}{(s+k_1)} + \frac{C}{(s+k_2)}$$

$$A = \frac{1}{(s+k_1)(s+k_2)}\bigg|_{s=0} = \frac{1}{k_1 k_2}$$

$$B = \frac{1}{s(s+k_2)}\bigg|_{s=-k_1} = \frac{1}{k_1^2 - k_1 k_2}$$

$$C = \frac{1}{s(s+k_1)}\bigg|_{s=-k_2} = \frac{1}{k_2^2 - k_1 k_2}.$$

$$y_2(t) = \frac{1}{k_1 k_2} + \frac{1}{k_1^2 - k_1 k_2} e^{-k_1 t} + \frac{1}{k_2^2 - k_1 k_2} e^{-k_2 t}$$

c). $\lim\limits_{t \to \infty} y_2(t) = \lim\limits_{s \to 0} sY_2(s)$

$\lim\limits_{t \to \infty} y_2(t) = \frac{1}{k_1 k_2}$, the other two terms decay to 0.

$$\lim_{s \to 0} sY_2(s) = \frac{\cancel{s}}{\cancel{s}(s+k_1)(s+k_2)}$$

$$= \frac{1}{(0+k_1)\cdot(0+k_2)} = \frac{1}{k_1 k_2}$$

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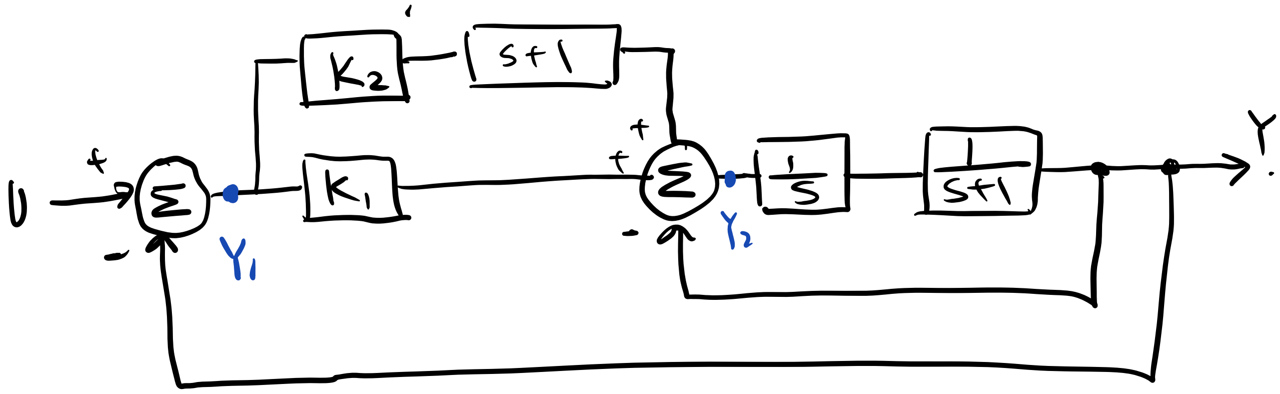
Problem 2



1. Equivalent Block Diagram

$$Y_1 = U - Y$$

$$Y_2 = Y_1 K_1 + (s+1) Y_1 K_2 - Y$$

$$= Y_1 (K_1 + (s+1)K_2) - Y.$$

$$= (U-Y)(K_1 + (s+1)K_2) - Y.$$

$$Y = Y_2 \left(\frac{1}{s(s+1)}\right)$$

$$= \left(\frac{1}{s(s+1)}\right) \{(K_1 + (s+1)k_2)(U-Y) - Y)$$

$$= \frac{1}{s(s+1)} \cdot (K_1 + (s+1)k_2) \cdot U$$

$$- \frac{1}{s(s+1)} \cdot (K_1 + (s+1)k_2) \cdot Y.$$

$$- \frac{1}{s(s+1)} \cdot Y.$$

$$Y \cdot \left(1 + \frac{1}{s(s+1)} + \frac{k_1+(s+1)k_2}{s(s+1)}\right) = \frac{k_1+(s+1)k_2}{s(s+1)} \cdot U.$$

$$Y \cdot \left(\frac{s^2+s+1+k_1+sk_2+k_2}{\cancel{s(s+1)}}\right) = U \frac{k_1+sk_2+k_2}{\cancel{s(s+1)}}$$

$$\frac{Y}{U} = \frac{k_1+sk_2+k_2}{s^2+(k_2+1)s+(k_1+k_2+1)}.$$

$$U \longrightarrow \boxed{\frac{k_1+sk_2+k_2}{s^2+(k_2+1)s+(k_1+k_2+1)}} \longrightarrow Y.$$

2. Stability.

Hurwitz Matrix. Deg (2)

$$M_H = \begin{bmatrix} k_2+1 & 1 \\ 0 & k_1+k_2+1 \end{bmatrix}$$

$Det_1 = k_2+1 > 0.$

$Det_2 : (k_2+1)(k_1+k_2+1) > 0.$

$$\therefore \begin{cases} k_2 > -1 \\ k_1 > 0. \end{cases}$$

3. $R(t) = 1(t) \leftarrow$ unit step

$$R(s) = \frac{1}{s}. \qquad \text{Error} = Y(s) - R(s).$$

$$\lim_{t \to \infty} E(t) = \lim_{s \to 0} sE(s)$$

$$Y(s) = \frac{k_1 + sk_2 + k_2}{s(s^2 + (k_2+1)s + (k_1+k_2+1))}.$$

$$E(s) = Y(s) - R(s) = \frac{k_1 + sk_2 + k_2}{s(s^2 + (k_2+1)s + (k_1+k_2+1))} - \frac{s^2 + (k_2+1)s + (k_1+k_2+1)}{s(s^2 + (k_2+1)s + (k_1+k_2+1))}.$$

$$= \frac{-s^2 + (\cancel{k_2 - k_2} - 1)s + (\cancel{k_1 + k_2 - k_1 - k_2} - 1)}{s(s^2 + (k_2+1)s + (k_1+k_2+1))}.$$

$$= -\frac{s^2 + s + 1}{s(s^2 + (k_2+1)s + (k_1+k_2+1))}.$$

$$\lim_{s \to 0} sE(s) = -\frac{\cancel{s^2 + s} + 1}{\cancel{s^2} + \cancel{(k_2+1)s} + (k_1+k_2+1)}.$$

$$= -\frac{1}{k_1+k_2+1}.$$

4. Derivative controller $D_d(s) = K_d\, s$.
Integral Controller $D_i(s) = \frac{1}{s} K_I$

- If k_1 became a derivative controller D_d,

Tracking error:

$$\lim_{s \to 0} -\frac{1}{K_d s + k_2 + 1} = -\frac{1}{k_2 + 1}$$

Does NOT make error zero.

- If k_1 became an integral controller D_i

Tracking error:

$$\lim_{s \to 0} -\frac{1}{\frac{1}{s} k_I + k_2 + 1} = -\frac{1}{\infty}$$

$$= 0.$$

CAN make error 0 (regardless of K_i)