Fall 2021
EE 134
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134 Graph Theory Applications
Final
Tuesday Oct 26, 2021

NAME: ____________________ **UID**: ____________________

This exam has 3 questions, for a total of 20 points.

Good luck!

Problem 1 (`6 points`) We are given a set of n nodes that can potentially all connect to each other over wireless. In particular, we assume that time is divided into timeslots, and during each timeslot, each node can send one packet to any other node. However, as is the case with most wireless nodes today, these nodes are half-duplex, namely, at each timeslot, a node can either transmit a packet or receive a packet, but not both. For example, if during timeslot 1 node 1 sends a packet to node 2, node 1 cannot during the same timeslot receive any packet, and node 2 cannot during the same timeslot 1 transmit any packet. We are also given a matrix C where element c_{ij} is an integer number (potentially zero) specifying how many packets node i wants to send to node j. You are asked to design, which node transmits during which time slot, so that all nodes convey their packets as fast as possible. Reduce this to a graph theory problem.

Problem 2 (6 points) An airline company provides flights connecting m origin-destination pairs (connecting city i to city j), offering flights from city i to city j y_{ij} times per week. The airline has a fleet of n planes, where each plane (and their crew) may serve several origin-destination flights in any given week. Because of Covid, the airline decides to reduce the number of flights, and decides to offer $x_{ij} \leq y_{ij}$ flights connecting city i to city j each week. The airline wants to offer these flights using a subset of its planes and ground the rest. Provide a graph theory problem formulation, that allows the airline company to maximize the number of planes it can ground (not use at all), while keeping each of the active planes on a subset of the flights they supported in the past.

Problem 3 (`8 points`): Which of the following statements are true and which not? Give a brief (1-2 sentences) justification or proof.

1. For any given graph G, the number of independent sets we can find in this graph equals the number of vertex covers we can find in the same graph.

2. Assume a graph has a cut vertex (a vertex whose removal disconnects the graph into two disjoint non-empty parts). Then this graph cannot have a Hamiltonian cycle.

3. If a graph G has exactly two vertices of odd degree, then these are connected by a path.

4. A tree can have two distinct perfect matchings (distinct means that they differ in at least one edge).