22W-EC ENGR-131A-LEC-1 Exam 1

MIKE QU

TOTAL POINTS

100 / 100

QUESTION 1

1 Problem 0: Academic Integrity Pledge 0 / 0

✓ **+ 0 pts** Correct

- 10 pts Missing

QUESTION 2

2 Problem 1 15 / 15

✓ **+ 7 pts** (a) Correct

✓ **+ 8 pts** (b) Correct

- 3 pts (a) Mistake

- 4 pts (b) Mistake

QUESTION 3

3 Problem 2 15 / 15

✓ **+ 7 pts** (a) Correct

✓ **+ 8 pts** (b) Correct

- 3 pts (a) Mistake

- 4 pts (b) Mistake

QUESTION 4

4 Problem 3 15 / 15

✓ **+ 8 pts** Numerator correct

✓ **+ 7 pts** Denominator correct

QUESTION 5

5 Problem 4 15 / 15

✓ **+ 3 pts** (a) Correct

✓ **+ 3 pts** (b) Correct

✓ **+ 3 pts** (c) Correct

✓ **+ 3 pts** (d) Correct

✓ **+ 3 pts** (e) Correct

- 1 pts 1 mistake

- 2 pts 2 mistake

- 3 pts 3 mistake

QUESTION 6

6 Problem 5 20 / 20

✓ **+ 4 pts** (a) Correct

✓ **+ 4 pts** (b) Correct

✓ **+ 4 pts** (c) Correct

✓ **+ 4 pts** (d) Correct

✓ **+ 4 pts** (e) Correct

QUESTION 7

7 Problem 6 20 / 20

✓ **+ 6 pts** (a) P(0) Correct

✓ **+ 6 pts** (a) P(1) Correct

✓ **+ 8 pts** (b) Correct

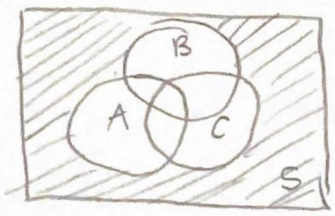
Mike Qu
603 536 630

0. Please read the Policy on the Academic Integrity on the first page of this exam and follow the instructions. I, Mike Qu with UID 603 536 630 have read and understood the policy on academic integrity
1. (15 pts) Consider three events A, B, and C defined on the same sample space S. Draw a Venn diagram and write the expression for the following events. Use only complements, intersections, and unions operations.

   (a) None of the events occur.

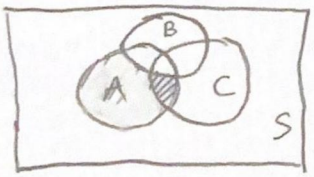
   (b) Events A and C occur, but B does not.

a)



$(A \cup B \cup C)^c$

b)



$(A \cap C \cap B^c)$

1 Problem 0: Academic Integrity Pledge **0 / 0**

✓ **+ 0 pts** **Correct**

- 10 pts Missing

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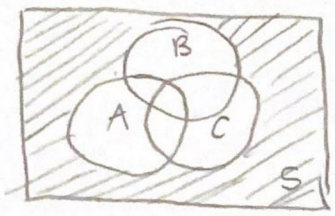
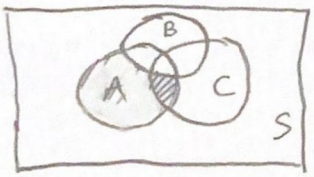
a)



$(A \cup B \cup C)^c$

b)



$(A \cap C \cap B^c)$

2 Problem 1 15 / 15

- ✓ **+ 7 pts** (a) Correct
- ✓ **+ 8 pts** (b) Correct
- **- 3 pts** (a) Mistake
- **- 4 pts** (b) Mistake

2. (15 pts) Suppose $P(A) = 1/3$, $P(A \cup B) = 2/3$, $P(A \cap B) = 1/4$. Compute:

(a) $P(B)$

(b) $P(A \cap B|B)$

Show all your work.

a) $P(A) + P(B) - P(A \cap B) = P(A \cup B)$

$\frac{1}{3} + P(B) - \frac{1}{4} = \frac{2}{3}$ multiply by 12.

$4 + 12P(B) - 3 = 8$

$12P(B) = 7$

$P(B) = \boxed{\frac{7}{12}}$

b) $P(A \cap B|B)$

$= \frac{P((A \cap B) \cap B)}{P(B)}$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{(\frac{1}{4})}{(\frac{7}{12})}$

$= \frac{1}{4} \cdot \frac{12}{7}^{3}$

$= \boxed{\frac{3}{7}}$

3 Problem 2 15 / 15

- ✓ **+ 7 pts** (a) Correct
- ✓ **+ 8 pts** (b) Correct
- **- 3 pts** (a) Mistake
- **- 4 pts** (b) Mistake

3. (15 pts) We draw 8 cards from an ordinary deck of 52 cards. What is the probability that we have exactly three queens?

4 total queens, choose any 3.

remaining 48 can be any non-queen card

$$\frac{\binom{4}{3} \cdot \binom{48}{5}}{\binom{52}{8}}$$

All combinations of choosing 8 cards out of 52

4 Problem 3 **15 / 15**

- ✓ **+ 8 pts** Numerator correct
- ✓ **+ 7 pts** Denominator correct

4. (15 pts) Mention True or False for each of the following questions.

Mentioning the correct answer is worth +3 points, mentioning the incorrect answer is worth −1 points. Leaving the answer blank is worth 0 points.

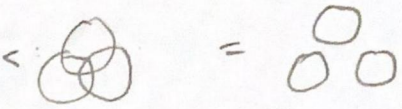
(a) For all events A, B, C, we have $P(A \cup B \cup C) \leq P(A) + P(B) + P(C)$.

(b) For a random variable X, $\text{VAR}(bX) = b^2\text{VAR}(X)$ for all real values of b.

(c) If $P(A \cap B) = 0$, then either $P(A)$ or $P(B)$ must be equal to zero.

(d) If events X and Y are mutually exclusive, then they are also independent.

(e) If random variables X and Y are independent and $Var[X] = a$, and $Var[Y] = b$. Then, $Var[X + Y] = a + b$.



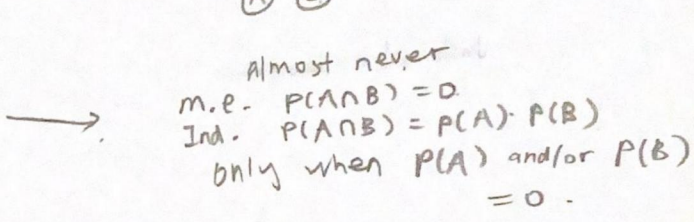
a) True

b) True

c) False

d) False

e) True



5 Problem 4 15 / 15

- ✓ + 3 pts (a) Correct
- ✓ + 3 pts (b) Correct
- ✓ + 3 pts (c) Correct
- ✓ + 3 pts (d) Correct
- ✓ + 3 pts (e) Correct
  - - 1 pts 1 mistake
  - - 2 pts 2 mistake
  - - 3 pts 3 mistake

5. (20 pts) Suppose X has the following PMF:

$$X = \begin{cases} -5, & \text{with probability } 1/5, \\ 0, & \text{with probability } 3/5, \\ 5, & \text{with probability } 1/5. \end{cases} \quad (1)$$

Show all your work.

(a) Compute the mean and variance of X.

(b) Compute the mean and variance of $Y = X^2$.

(c) Compute $\mathbf{E}[XY]$.

(d) Are X and Y uncorrelated? Justify your answer.

(e) Are X and Y independent? Justify your answer.

$Y: \begin{cases} 25 & p = \frac{2}{5} \\ 0 & p = \frac{3}{5} \end{cases}$

$X \cdot Y \begin{cases} -125 & p = \frac{2}{25} \\ 0 & p = \frac{21}{25} \\ 125 & p = \frac{2}{25} \end{cases}$

a) $m_X = \mathbb{E}[X] = -5 \cdot \frac{1}{5} + 0 \cdot \frac{3}{5} + 5 \cdot \frac{1}{5}$

$= -1 + 0 + 1 = \boxed{0}$

$\text{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$

$\mathbb{E}[X^2] = (-5)^2 \cdot \frac{1}{5} + 0 \cdot \frac{3}{5} + (5)^2 \cdot \frac{1}{5}$

$= 5 + 5 = 10$

$\text{Var}(X) = 10 - 0^2 = \boxed{10}$

b) $\mathbb{E}[Y] = \mathbb{E}[X^2] = \boxed{10}$ (calculated in part a)

$\mathbb{E}[Y^2] = 25^2 \cdot \frac{2}{5} + 0 \cdot \frac{3}{5} = 250$

$\text{Var}(Y) = 250 - 10^2 = \boxed{150}$

c) $\mathbb{E}[XY] = -125 \cdot \frac{2}{25} + 0 \cdot \frac{21}{25} + 125 \cdot \frac{2}{25} = \boxed{0}$

d) $\mathbb{E}[X \cdot Y] = \mathbb{E}[X] \cdot \mathbb{E}[Y]$.

$0 = 0 \cdot 10$ ✓

They are uncorrelated

e) $Y = X^2$

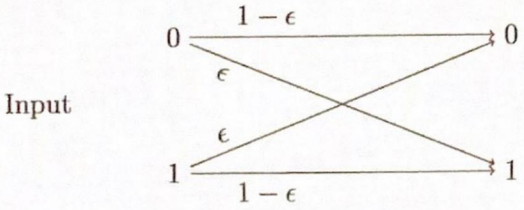
Because Y depends on the value X takes,

They are NOT independent

6 Problem 5 20 / 20

- ✓ + 4 pts (a) Correct
- ✓ + 4 pts (b) Correct
- ✓ + 4 pts (c) Correct
- ✓ + 4 pts (d) Correct
- ✓ + 4 pts (e) Correct

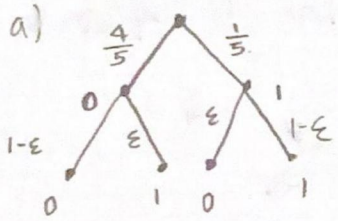
6. (20 pts) A binary symmetric communication channel is shown in the figure. The probability that the output is 1 given that the input is 0 is ϵ. Similarly, probability that the output is 0 given that the input is 1 is ϵ. Assume that input symbols 0 and 1 are chosen for transmission with probabilities $\frac{4}{5}$ and $\frac{1}{5}$, respectively.



(a) Calculate the probability of each output.

(b) Given that the output was 1, what is the probability that the output was flipped by the channel?

a)



Total Prob. $\quad$ out=1 | flip $\quad$ out=1 | no flip

$$P(\text{out}=1) = \frac{4}{5}\cdot\varepsilon + \frac{1}{5}(1-\varepsilon) = \boxed{\frac{1}{5} + \frac{3}{5}\varepsilon}$$

$$P(\text{out}=0) = \frac{4}{5}\cdot(1-\varepsilon) + \frac{1}{5}\varepsilon = \boxed{\frac{4}{5} - \frac{3}{5}\varepsilon}$$

b) $P(\text{flip} \mid \text{out}=1)$

Bayes rule

$$= \frac{P(\text{out}=1 \mid \text{flip}) \cdot P(\text{flip})}{P(\text{out}=1)}$$

$$= \frac{\frac{4}{5}\cdot\varepsilon}{\frac{1}{5} + \frac{3}{5}\varepsilon}$$

$$= \frac{4\varepsilon}{1+3\varepsilon}$$

7 Problem 6 20 / 20

- ✓ + 6 pts (a) P(0) Correct
- ✓ + 6 pts (a) P(1) Correct
- ✓ + 8 pts (b) Correct