

You have 3 hours to submit your work **directly on Gradescope under the Final Exam submission link.**
Please read and carefully follow all the instructions.

Instructions

- The exam is accessible from 10 am PST on March 19th to 10 am PST on March 20th. Once you open the exam, you will have 3 hours to upload your work (therefore open the exam at least 3 hours before the closing time).
- This exam is open book, open notes. You are allowed to consult your own class notes (homework, discussion, lecture notes, textbook). You are not allowed to consult with each other or solicit external sources for help (e.g., an online forum).
- For each question, start a new sheet of paper. Therefore, the number of pages of your scan should be at least the number of questions. It is ok to write multiple parts of a question on one sheet. Properly erase or cross out any scratch work that is not part of the answer.
- Please submit your exam through the corresponding submission link on Gradescope.
- Make sure to include your **full name** and **UID** in your submitted file.
- Make sure to **show all your work**. Unjustified answers will be at a risk of losing points.
- **Policy on the Academic Integrity**
During this exam, you are **disallowed** to contact with a fellow student or with anyone outside the class who can offer a solution e.g., web forum.
Please write the following statement on the first page of your answer sheet.
You will **lose 10 points** if we can not find this statement.

I YourName with UID have read and understood the policy on academic integrity.

1. (10 pts) You have two coins. One of them is a fair coin and the other has heads on both sides. You select a coin at random and toss it.
 - (a) What is the probability that the toss results in a heads?
 - (b) What is the probability that you had selected the fair coin given that the toss resulted in a heads?
 - (c) Suppose you toss the selected coin n times. What is the probability that you had selected the fair coin given that each of the n tosses result in a heads? Write your answer in terms of n .
2. (10 pts) Let X_1 and X_2 be two independent Bernoulli random variables with parameters p_1 and p_2 respectively.

- (a) Find the characteristic function of $Y = 3X_1 + 5X_2$.
- (b) Using the characteristic function from part (a), find $E[Y]$.

3. (10 pts) State the weak law of large numbers.
4. (15 pts) Consider the jointly Gaussian random variables X and Y that have the following joint PDF:

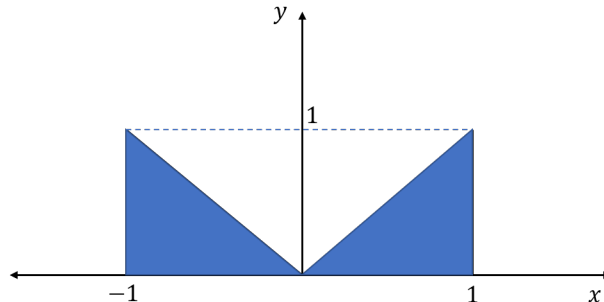
$$f_{X,Y}(x,y) = \frac{5}{2\pi} \exp \left\{ - \left[\frac{25}{32}x^2 - \frac{15}{4}xy + \frac{25}{2}y^2 \right] \right\}$$

- (a) Find $E[X]$, $E[Y]$, $VAR[X]$, $VAR[Y]$, and $COV[X, Y]$
 - (b) Find the marginal PDF of Y , $f_Y(y)$
 - (c) Find $f_{X|Y}(x|y)$
 - (d) Find $E[X|Y = y]$ and $E[X|Y]$
5. (10 pts) Suppose X is a Gaussian RV with mean 0 and variance 16. Let RV Y be defined as follows:

$$Y = \begin{cases} X, & \text{if } X \geq 1, \\ 2 - X, & \text{if } X < 1. \end{cases}$$

Compute the pdf of Y .

6. (15 pts) Let X and Y be continuous random variables that are uniformly distributed on the shaded area in the following figure:



- (a) Find the joint PDF of X and Y .
 - (b) Compute the marginal PDFs of X and Y .
 - (c) Find the conditional pdf of X given Y .
 - (d) Find $E[X|Y]$
7. (15 pts) Let X and Y be continuous random variables with the following joint PDF:

$$f_{X,Y}(x, y) = \begin{cases} k(x + y) & 0 \leq y \leq x < 1, \\ 0 & \text{otherwise,} \end{cases}$$

Let $Z = X + Y$.

- (a) Find the constant k .
 - (b) Are X and Y independent? Explain your answer.
 - (c) Find the PDF of Z .
8. (15 pts) A sequence of 1000 bits are transmitted over a channel. Each bit gets flipped independently of others with a probability 0.2. Let X be the total number of bit flips that occur.
- (a) What type of random variable is X ?
 - (b) Find $P(230 < X < 370)$. You can leave the final answer as a summation or integral.
 - (c) Estimate (approximate) the probability in part (a) using the central limit theorem. You can leave your answer in terms of the Q function.