EE 113 Digital Signal Processing

Spring 2017 Quiz-1

Closed Book

Name: SOLUTIONS

Student ID No.:__________________________

1) ________ / 10

2) ________ /10

3) ________ / 10

4) ________ / 20

TOTAL ________ / 50

TABLE 14.1 Properties of the DTFT.

	Sequence	DTFT	Property
1.	$x(n)$	$X(e^{j\omega})$	(14.1)
2.	$y(n)$	$Y(e^{j\omega})$	(14.2)
3.	$ax(n)+by(n)$	$aX(e^{j\omega})+bY(e^{j\omega})$	linearity
4.	$x(n-n_0)$	$e^{-j\omega n_0}X(e^{j\omega})$	time-shifts
5.	$e^{j\omega_o n}x(n)$	$X(e^{j(\omega-\omega_o)})$	frequency shifts
6.	$\cos(\omega_o n)x(n)$ $\sin(\omega_o n)x(n)$	$\frac{1}{2}X\left(e^{j(\omega-\omega_o)}\right)+\frac{1}{2}X\left(e^{j(\omega+\omega_o)}\right)$ $\frac{1}{2j}X\left(e^{j(\omega-\omega_o)}\right)-\frac{1}{2j}X\left(e^{j(\omega+\omega_o)}\right)$	modulation
7.	$x(-n)$	$X(e^{-j\omega})$	time-reversal
8.	$nx(n)$	$j\,\frac{dX(e^{j\omega})}{dw}$	linear modulation
9.	$x(n)\star y(n)$	$X(e^{j\omega})Y(e^{j\omega})$	convolution
10.	$x(n)y(n)$	$X(e^{j\omega})\circ Y(e^{j\omega})$	multiplication
11.	$x^*(n)$	$\left[X(e^{-j\omega})\right]^*$	conjugation

TABLE 20.1 Several properties of the Fourier Transform.

signal	Fourier Transform	property
$x(t)$	$X(j\Omega)$	
$y(t)$	$Y(j\Omega)$	
$ax(t)+by(t)$	$aX(j\Omega)+bY(j\Omega)$	linearity
$x(at)$	$\frac{1}{\lvert a\rvert}X(\frac{j\Omega}{a})$	scaling
$X(t)$	$2\pi\cdot x(-j\Omega)$	duality
$x(t-t_o)$	$e^{-j\Omega t_o}X(j\Omega)$	time-shifts
$e^{j\Omega_o t}x(t)$	$X(j\Omega-j\Omega_o)$	modulation
$\int_{-\infty}^{t}x(\tau)d\tau$	$\frac{1}{j\Omega}X(j\Omega)+\pi\cdot X(0)\cdot\delta(j\Omega)$	integration
$\frac{dx(t)}{dt}$	$j\Omega\cdot X(j\Omega)$	differentiation
$\int_{-\infty}^{\infty}x(\lambda)y(t-\lambda)d\lambda$	$X(j\Omega)Y(j\Omega)$	convolution
$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\lambda)Y(j\Omega-j\lambda)d\lambda$	multiplication
	$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(j\Omega)\rvert^2d\Omega$	Parseval's relation

TABLE 13.1 Some useful DTFT pairs over the interval $\omega\in[-\pi,\pi]$.

Sequence $x(n)$	DTFT $X(e^{j\omega})$ over one period
$x(n)=\delta(n)$	$X(e^{j\omega})=1$
$x(n)=\begin{cases}1, & 0\le n\le L-1\\ 0, & \text{otherwise}\end{cases}$	$X(e^{j\omega})=\begin{cases}L, & \omega=0\\ e^{-j\omega\frac{(L-1)}{2}}\cdot\frac{\sin(\omega L/2)}{\sin(\omega/2)}, & \text{otherwise}\end{cases}$
$x(n)=\alpha^n u(n),\ \lvert\alpha\rvert<1$	$X(e^{j\omega})=\frac{1}{1-\alpha e^{-j\omega}}$
$x(n)=-\alpha^n u(-n-1),\ \lvert\alpha\rvert>1$	$X(e^{j\omega})=\frac{1}{1-\alpha e^{-j\omega}}$
$x(n)=\frac{\omega_c}{\pi}\text{sinc}(\omega_c n)$	$X(e^{j\omega})=\begin{cases}1, & \lvert w\rvert<w_c\\ 0, & w_c\le\lvert w\rvert\le\pi\end{cases}$
$x(n)=e^{j\omega_o n}$	$X(e^{j\omega})=2\pi\delta(\omega-\omega_o)$
$x(n)=\cos(\omega_o n),\ \omega_o\in[-\pi,\pi]$	$X(e^{j\omega})=\pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$
$x(n)=\sin(\omega_o n),\ \omega_o\in[-\pi,\pi]$	$X(e^{j\omega})=-j\pi\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$

10 pt **Problem 1:**

The Nyquist rate of *x(t)* is 10 radians/s. What is the nyquist rate for each of the following. Please explain your answer. No points will be given without an explanation

(a) y(t) = *x(t)*x(t)*

(b) $y(t) = \text{sinc}\left(\frac{\Omega_o}{2}t\right)x(t)$ where $\Omega_o = 2 rads/\sec$

(a) $y(t) = x(t) * x(t) \Rightarrow Y(\Omega) = X(\Omega) \cdot X(\Omega)$

$\Rightarrow$ freq range of $X(f)$ & $Y(f)$ are identical

$\Rightarrow$ Nyquist freq for $y(t)$ is the same as for $x(t) = 10$ rads/sec

(b) $y(t) = \underbrace{\text{sinc}\left(\frac{\Omega_o}{2}t\right)}_{a(t)} x(t)$

$$A(\Omega) = \begin{cases} 1 & |\Omega| \leq 1 \text{ rads/s} \\ 0 & \text{otherwise} \end{cases}$$

$Y(\Omega) = A(\Omega) * X(\Omega) \Rightarrow$ freq span of $Y(\Omega)$ is

The sum of the freq span of $A(\Omega)$ & $X(\Omega)$ =

$5 + 1 = 6$ rads/s

Nyquist freq of $Y(\Omega)$ is $12 \frac{rads}{sec}$

10 pt **Problem 2:**

Find the DTFT of the following functions. Please be sure to show your work as no points will be given for simply stating the final answer.

(a) $x(n) = \cos\left(\frac{\pi}{3}n\right)\sin\left(\frac{\pi}{3}n\right)$

(b) $x(n) = \left(\frac{1}{2}\right)^{3n+2} u(n-3)$

(a) $x(n) = \frac{1}{2}\left\{e^{j\frac{\pi}{3}n} + e^{-j\frac{\pi}{3}n}\right\} \frac{1}{2j}\left\{e^{j\frac{\pi}{3}n} - e^{-j\frac{\pi}{3}n}\right\}$

$= \frac{1}{4j}\left\{e^{j\frac{2\pi}{3}n} - 1 + 1 - e^{-j\frac{2\pi}{3}n}\right\}$

$= \frac{1}{2}\sin\left(\frac{2\pi}{3}n\right)$

$X(e^{j\omega}) = \frac{\pi}{2j}\left[\delta\left(\omega - \frac{2\pi}{3}\right) - \delta\left(\omega + \frac{2\pi}{3}\right)\right]$

(b) $x(n) = \left(\frac{1}{2}\right)^2\left(\frac{1}{8}\right)^n u(n-3)$

$= \frac{1}{4}\left(\frac{1}{8}\right)^n u(n) - \frac{1}{4}\left(\frac{1}{8}\right)^0\delta(n) - \frac{1}{4}\left(\frac{1}{8}\right)\delta(n-1) - \frac{1}{4}\left(\frac{1}{8}\right)^2\delta(n-2)$

$X(e^{jw}) = \frac{1}{4}\left\{\frac{1}{1-\frac{1}{8}e^{-jw}} - 1 - \frac{1}{8}e^{-jw} - \frac{1}{64}e^{-j2w}\right.$

$= \frac{1}{4}\left\{\frac{1 - 1 + \frac{1}{8}e^{-jw} - \frac{1}{8}e^{-jw} + \frac{1}{64}e^{-j2w} - \frac{1}{64}e^{-j2w} + \frac{1}{512}e^{-j3w}}{1 - \frac{1}{8}e^{-jw}}\right\}$

$= \frac{1}{4}\frac{\frac{1}{512}e^{-j3w}}{1-\frac{1}{8}e^{-jw}} = \frac{1}{2048}\frac{e^{-j3w}}{1-\frac{1}{8}e^{-jw}}$

10 pt **Problem 3:**

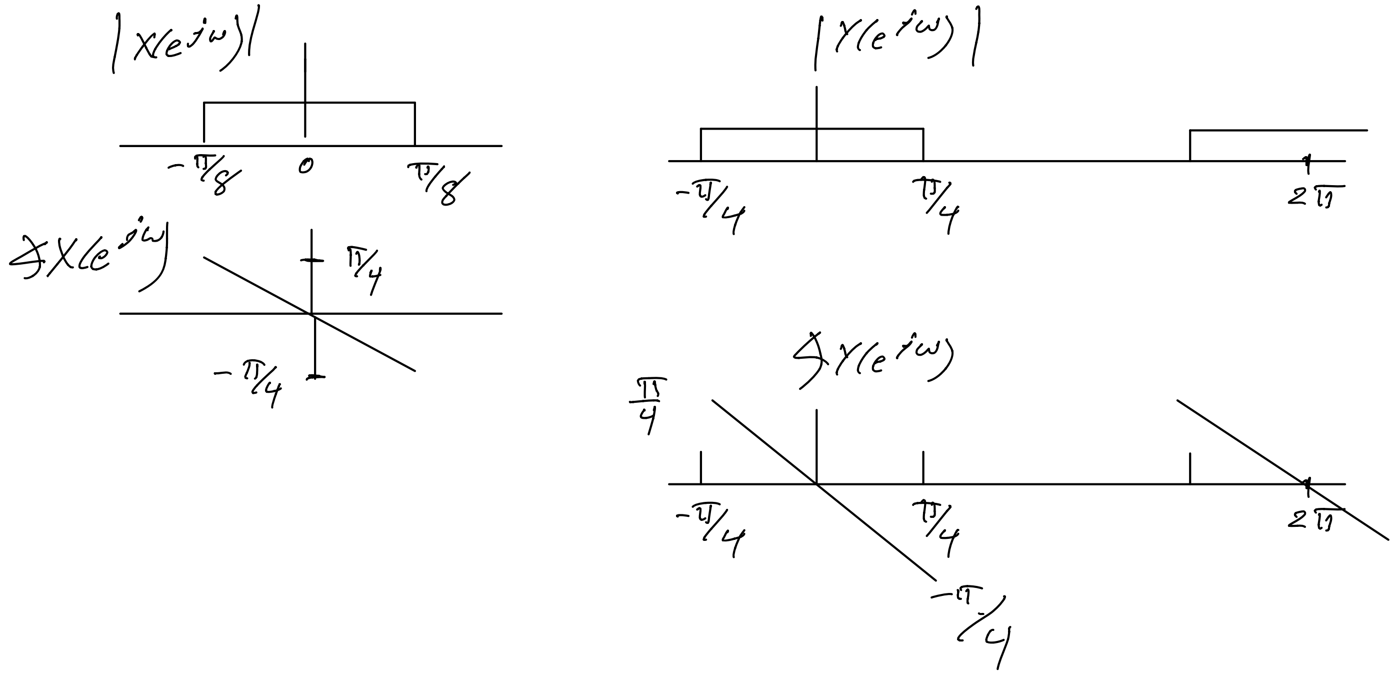
Find the DTFT of y(n) and then sketch its magnitude and phase response. Be sure to show your work.

$$y(n) = x(2n)$$

$$\text{where } x(n) = \frac{\sin\left(\frac{\pi}{8}(n-2)\right)}{(n-2)}$$

$$X(e^{j\omega}) = \begin{cases} \pi e^{-j2\omega} & |\omega| < \frac{\pi}{8} \\ 0 & \text{otherwise} \end{cases}$$

$$Y(e^{j\omega}) = \frac{1}{2}\sum_{\ell=0}^{1} X\left(e^{j\left(\frac{\omega}{2} + \frac{2\pi}{2}\ell\right)}\right) = \frac{1}{2}\left[X\left(e^{j\omega/2}\right) + X\left(e^{j\left(\frac{\omega}{2} - \pi\right)}\right)\right]$$



20 pt **Problem 4:**

We start with an analog signal x(t) given below

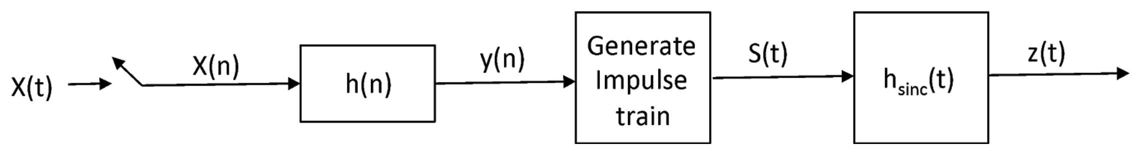
$$x(t) = \cos(2\pi\, 150\, t) + 0.3\sin(2\pi\, 1900\, t) + 0.7\cos(2\pi\, 3300\, t)$$

x(t) is then sampled at a sampling frequency of 4000 Hz to generate the sequence {x(n)}

The sequence {x(n)} is then filtered by a brick wall (ideal) Low Pass Filter with a cutoff frequency $w_c = \frac{\pi}{2}$ to generate the sequence y(n)

The sequence y(n) is then used to create the analog pulse train $s(t) = \sum y(n)\,\delta\left(t - n\frac{1}{4000}t\right)$

S(t) in turn is passed through an analog filter with impulse response $h_{sinc}(t) = \operatorname{sinc}(\pi\, 4000\, t)$ to produce the analog signal z(t)



(a) give an expression for x(n)

(b) sketch the magnitude response of $X(e^{jw})$ from –pi to pi

(c) give an expression for y(n)

(d) sketch the magnitude response of S(f)

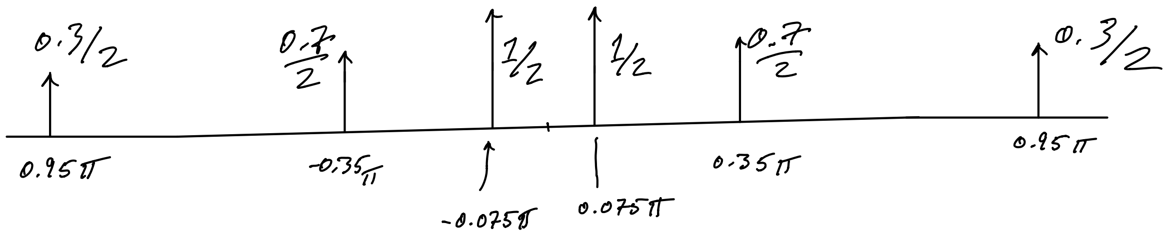
(e) give an expression for z(t)

(a) $x(n) = \cos\left(2\pi \frac{150}{4000} n\right) + 0.3 \sin\left(2\pi \frac{1900}{4000} n\right) + 0.7 \cos\left(2\pi \frac{3300}{4000} n\right)$

$x(n) = \cos\left(2\pi \frac{150}{4000} n\right) + 0.3 \sin\left(2\pi \frac{1900}{4000} n\right) + 0.7 \cos\left(2\pi \frac{4000-3300}{4000} n\right)$

(b) $x(n) = \cos\left(2\pi \frac{150}{4000} n\right) - 0.3 \sin\left(2\pi \frac{1900}{4000} n\right) + 0.7 \cos\left(2\pi \frac{700}{4000} n\right)$

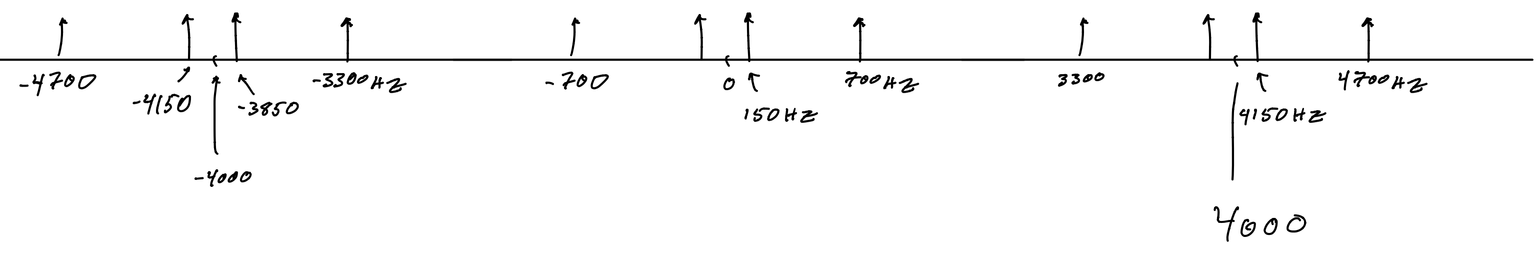
$w = \frac{150}{4000} 2\pi = 0.075\pi$ $w = \frac{1900}{4000} 2\pi = 0.95\pi$ $w = \frac{700}{4000} 2\pi = 0.35\pi$



(c) since w_c for $h(n)$ is $\frac{\pi}{2}$ the term corresponding to $w = 0.95\,\pi$ will be filtered out & thus $y(n)$ will only contain the 1st and last terms of $x(n)$

$$y(n) = \cos\left(2\pi \frac{150}{4000} n\right) + 0.7 \cos\left(2\pi \frac{700}{4000} n\right) = \cos\left(\frac{3\pi}{40} n\right) + 0.7 \cos\left(\frac{7\pi n}{20}\right)$$

(d)



(e) $z(t) = \cos(2\pi\, 150 t) + 0.7 \cos(2\pi\, 700 t)$